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Strategies analysis of luxury fashion rental platform in sharing economy

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ABSTRACT

As the sharing economy has grown rapidly in recent years, luxury fashion rental has become a prominent business trend. An increasing number of designer brands are finding ways to co-operate with rental platforms rather than compete with them. We develop a stylized model to study the impact of business-to-consumer product sharing on luxury fashion brands. Specifically, we consider a setting where a designer brand firm provides a luxury fashion product to a rental platform, which can either purchase the product from the firm then rent to consumers, or allow the designer brand firm to use its platform to reach the consumers directly with a commission fee. Our analysis reveals that the presence of a rental platform leads to two simultaneous effects: a market expansion effect and a cannibalization effect. In the base model, we show that the market expansion effect dominates the cannibalization effect. Therefore, the designer brand firm can benefit from the appearance of a rental platform. Further, our analysis reveals that the optimal choice between the wholesale and agency contract of the firm and the platform depends critically on two parameters: the revenue-sharing proportion and salvage value. We illustrate that when the revenue-sharing proportion is relatively large and salvage value is relatively small, both the firm and the platform benefit more under the agency contract. Moreover, we illustrate the robustness of our main insights to consider the presence of different consumer segments, consumers' conspicuous behavior, competition between rental platforms, and two-period rental setting.

1. Introduction

As the sharing economy has grown rapidly in recent years, luxury-rental-on-demand has become a hot trend, mainly driven by young consumers craving newness and novelty while embracing consumption sustainability (Imran and Anita, 2019). With more and more companies entering the luxury fashion rental market, consumers prefer to rent rather than purchase products like designer fashion clothes (e.g., Rent the Runway, Chic by Choice, LeTote), luxury fashion (e.g., Armarium), fine jewelry (e.g., Switch), and designer handbags (e.g., Bag Borrow or Steal). Even some traditional retailers, such as Ann Taylor and Bloomingdale's, have launched rental services successively, while other brands like J. Crew, Levi's, and Club Monaco actively seek cooperation with rental platforms (Elizabeth, 2018a). According to The *Wall Street Journal*, GlobalData Retail estimates that the valuation of the fashion rental market

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in 2018 is $1 billion and predicts that it will grow to $2.5 billion in 2023 (Khadeeja, 2019).

Traditionally, rental platforms are in competition with designer brand firms and retailers. For example, when Urban Outfitters launched its own rental service, it expected some cannibalization of its retail sales. “We do anticipate that some sales might be impacted,” said David Hayne, Urban Outfitters’ chief digital officer (Kate, 2019). However, the fashion rental business has been very well accepted by designer brand firms and retailers. Some designer brand firms actively cooperate with rental platforms. As stated by Jennifer Hyman, co-founder and chief executive of Rent the Runway, the way rental platforms work with designers has changed theatrically in the past few years. Platforms work with fashion designers or retailers to help them launch their own rental services or let them use the rental platform to reach consumers directly (Imran and Anita, 2019). Motivated by this emerging business practice, our research aims to answer the following research questions. First, in the luxury fashion industry, should designer brand firms cooperate with rental platforms? Second, under cooperation with rental platforms, should the firm and the platform adopt a wholesale contract or an agency contract? Third, how does the rental business affect consumer surplus? The answers to these questions will provide relevant managerial insights to the managers in related industries.

To address these questions, we develop a stylized model consisting of a rental platform, a designer brand firm, and a unit mass of consumers. The upstream designer brand firm can either sell fashion products directly to the consumer or use a platform to rent the product. We consider two prevailing contracts commonly used in the luxury fashion rental market: the wholesale contract and the agency contract. First, we study the wholesale contract, under which the firm determines a wholesale price charged to the platform and a retail price charged to consumers, and the platform determines the rental price. Next, we analyze the agency contract, under which the firm sets both the retail price and the rental price, while the platform acts as an agency and collects a part of the rental profit as a commission fee. Further, we compare the benefits of the firm and the platform under different contracts and analyze the factors that influence the contract choice of the agency contract and the wholesale contract.

Our analysis highlights three main findings. First, the presence of a rental platform can benefit the designer brand firm. This is due to the tradeoff between the market expansion effect and the cannibalization effect caused by the rental market. On the one hand, the rental platform offers consumers an alternative to purchase the product, causing a decrease in the demand of the retail market, i.e., *cannibalization effect*. On the other hand, some low-valuation consumers who do not purchase the product in the retail market may choose to rent it, leading to increased total market demand, i.e., *market expansion effect* (*expansion effect* hereafter). Our analysis shows that under either a wholesale contract or an agency contract, the expansion effect always dominates the cannibalization effect, and consequently, the designer brand firm earns a higher profit with the presence of a rental platform. This also explains why designer brand firms are willing to work with rental platforms.

Second, the firm favors the agency contract when the proportion of the rental market revenue that a platform can keep is relatively small and the salvage value of returned rental products is large, while the platform favors the agency contract when the salvage value falls into a certain range. The optimal choice between the agency and wholesale contracts mainly relies on the revenue-sharing proportion and the salvage value, and we illustrate that when the proportion is comparatively large and the salvage value is comparatively small, both the firm and the platform benefit more from the agency contract than wholesale contract, which implies that under this scenario, the agency contract can improve the overall profit of the supply chain.

Third, we find that compared with the wholesale contract and the single traditional channel, the retail price under the agency contract is strictly lower, which indicates that when the firm has the power to determine the rental price under the agency contract, the equilibrium retail price of the traditional channel will decrease. For high valuation consumers who purchase the product, their consumer surplus will increase and benefit more from the presence of a rental platform under the agency contract.

We also illustrate the robustness of the base model by considering the presence of different consumer segments, consumers’ conspicuous behavior, competition between rental platforms, and two-period rental setting. In a nutshell, we find that our core insights are robust and these extensions also provide some interesting new insights. When considering a subset of consumers who always prefer renting the product to purchasing it, we find that the designer brand firm may not always benefit from the rental market as in the base model. When considering consumers who buy the product because they desire uniqueness, we find that conspicuous behavior will reduce the Pareto-improving region in which both the firm and the platform are willing to choose the agency contract rather than the wholesale contract. When considering the competition between two rental platforms, we find that both firms will choose the same contract form in equilibrium. When extending our base model to the two-period rental setting, we find that the designer brand firm always prefers low price difference strategy under the wholesale model, while under the agency model, the designer brand firm will strategically choose between low price difference strategy and high price difference strategy.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature. The model setup is described in Section 3. Next, we present the equilibrium results and main findings in Section 4. Further, we consider several model extensions in Section 5. In the last section, we summarize this study and provide some future research directions. All proofs are provided in Appendix A.

2. Literature review

This study is related to three research streams: sharing economy, sell-or-lease strategy, and agency pricing. Recently, the rapidly developing sharing economy has attracted extensive attention from researchers. Belk (2014) conceptually depicts the sharing economy phenomenon in the internet era and provides some implications for business operations. Jiang and Tian (2018) analytically examine the influence of product sharing between strategic consumers. They discover that, depending on the marginal cost, product sharing may be win-win or lose-lose for the platform and consumers. Tian and Jiang (2018) further investigate the effect of product sharing on the distribution channel and reveal that the sharing among consumers benefit the retailer more than the manufacturer.

Benjaafar et al. (2018) conduct a similar model considering peer-to-peer sharing. They prove that usage levels and ownership can be either lower or higher compared to a scenario without product sharing, depending on the rental price. Wang et al. (2020) provide a product-sharing model considering the participation of the firm in the sharing market. They divide consumers into two types, high and low usage, and show that the firm will have an incentive to rent the product only when the joining cost is comparatively low and the proportion of consumers with low usage is comparatively high.

Most of the present empirical and analytical studies focus on sharing business practice in different industries, including ride-sharing industry (e.g., Bellos et al., 2017; Najmi et al., 2017; Albiński et al., 2018; Yu et al., 2019), hospitality industry (e.g., Zervas et al., 2017; Roma et al., 2019), food industry (e.g., Choi et al., 2019), and healthcare industry (e.g., Asghari and Al-e 2020). For the fashion industry, Choi and He (2019) conduct an analytical model to study the impact of peer-to-peer collaborative consumption (P2P-CC) and the operation strategies of the platform. They show that P2P-CC is always beneficial for the fashion brand and the consumers. Furthermore, they prove that the platform will benefit more from adopting revenue sharing pricing than adopting fixed service pricing. Our paper is closely related to Yuan and Shen (2019), which also focuses on business-to-consumer product sharing. They investigate the impact of consumers' illegal renting behavior on the fashion retailing market and indicate that the retailer can prevent illegal renting behavior by increasing the product return cost. For the system performance, they show that the centralized system is better for the retailer and the renter. Instead of assuming that the platform rents the product and the firm sells the product separately, we study the impact of business-to-consumer product sharing on a distribution channel and focus on the contract choice between the designer brand firm and the rental platform.

Our work further complements the stream of literature on the firm's sell-or-lease strategy decision (e.g., Bagnoli et al., 1989; Desai and Purohit, 1998; Desai and Purohit, 1999; Bhaskaran and Gilbert, 2005; Agrawal et al., 2012). Recently, Dou et al. (2017) study a monopoly firm's leasing or selling strategy for an information product, considering two types of consumer valuation depreciation. They discover that leasing benefits more than selling for vintage depreciation information products. However, when the extent of individual depreciation is above a certain threshold, selling will become more profitable than leasing for the information product with individual depreciation. Instead of focusing on a single party's sell-or-lease strategy as previous studies, our research focuses on the distribution channel's sell-or-lease strategy in a marketplace where the platform can rent out their purchased product. In this scenario, the platform, which can be regarded as a consumer of the firm, is also the firm's indirect competitor.

Our paper also complements the stream of literature on agency pricing. The early research on agency pricing focuses on the industry of e-book. Hao and Fan (2014) conduct a horizontally differentiated model to study the pricing strategy of e-books. They find that compared with a wholesale contract, e-book prices are higher under the agency contract and the publisher is worse off. Tan and Carrillo (2017) extend this research by considering a vertically differentiated model. They discover that the agency contract can outperform the wholesale contract for both the publisher and the retailer, and further ascertain that this benefit is mainly due to the publisher's power of setting the price. Recent research focuses on a broader context. Hao et al. (2017) reveal that an app's price will be higher under an agency advertising contract than under traditional advertising. Abhishek et al. (2016) discover that the impact of an electronic channel's sale on the traditional channel's demand plays a crucial role in the retailer's choice between agency selling and reselling. Tian et al. (2018) consider the impact of competition between upstream suppliers and prove that the reselling mode will outperform the agency selling mode when the competition intensity between upstream suppliers is high and the order-fulfillment costs is large. Shen et al. (2019) consider a scenario in which the manufacturer also needs to pay for slotting in addition to paying a proportion to the platform. They show that the agency model can be win-win when the manufacturer and the platform can negotiate both the slotting fee and the revenue-sharing proportion. We contribute to this literature stream by investigating the agency pricing model in the luxury fashion rental industry.

3. Model setting

We consider a luxury fashion retail market consisting of a designer brand firm ("the firm") who sells a luxury product, a rental platform ("the platform") that rents the product, and a continuum of consumers. Table 1 summarized the notations used in this study. We use subscripts to represent different parties: the designer brand firm (denoted by F), the rental platform (denoted by P), and the supply chain (denoted by SC). We use superscripts to represent different channel structures and contracts including the single traditional channel (denoted by N), vertically integrated channel model (denoted by I), wholesale model (denoted by W), and agency model (denoted by A).

3.1. Consumers

Consumers have a heterogeneous valuation of the product. We normalize the total market size to 1, and assume consumers' valuation of the product, v, is uniformly distributed between 0 and 1 (i.e., $v\ U[0, 1]$). Consumers' utility from buying the product is $u_b = v - p_b$, in which the subscript b denotes the retail market and p_b denotes the retail price charged by the designer brand firm. When there is no rental platform, consumers will purchase the product only when their valuation is higher than the retail price. We define the threshold $v_b = p_b$ as the level where consumers are indifferent about buying the product or not. Correspondingly, the demand of the retail market becomes $D_b^N = \Pr(v - p_b \geq 0) = 1 - p_b$.

Similar to Yuan and Shen (2019), the consumer's valuation of the rental product is λv, where $\lambda < 1$ represents the discounted value of the rental product. Note that a consumer can enjoy the purchased product indefinitely while they can only use the rental product for a pre-determined period. As a result, λ is related to the length of time that a consumer can enjoy the rental product.

Table 1
Summary of Notations.

Notation	Definition
v	Consumers' valuation of the product, $v \ U[0, 1]$
λ	The discounted value of the rental product, $0 < \lambda < 1$
D_b	Demand of retail market
D_r	Demand of rental market
p_b^k	The unit retail price, decision variable ($k = N, I, W, A$)
p_r^k	The unit rental price, decision variable ($k = N, I, W, A$)
ϵ	The unit salvage value of the returned product
w	Wholesale price charged by the firm, decision variable
α	Proportion of the revenue from the rental market that the platform charges as a commission fee, $0 < \alpha < 1/2$
v_b	Indifferent valuation point at which consumers are indifferent about buying the product or not
v_r	Indifferent valuation point at which consumers are indifferent about renting the product or not
v_b^r	Indifferent valuation point at which consumers are indifferent about the decision to purchase the product or rent it
π_i^k	Profit of each party ($i = F, P, SC$) under different models ($k = N, I, W, A$)
CS^k	Consumer surplus under different models ($k = N, I, W, A$)
SW^k	Social welfare under different models ($k = N, I, W, A$)

Specifically, a higher value of λ suggests that the rental product has a relatively longer rental term and vice versa. The consumer's utility from renting the product is $u_r = \lambda v - p_r$, in which the subscript r denotes the rental market. When only the rental platform exists, consumers will rent the product when $\lambda v - p_r \geq 0$. Similarly, we define the threshold $v_r = \frac{p_r}{\lambda}$ as the level where consumers are indifferent about renting the product or not.

We characterize the threshold $v_b^r = \frac{p_b - p_r}{1-\lambda}$ as the indifferent valuation point at which consumers are indifferent about the decision to purchase the product or rent it. When $\frac{p_r}{\lambda} < p_b$, it can be readily shown that $v_r < v_b < v_b^r$. In this case, as shown in Fig. 1, consumers can be divided into three groups: (a) Consumers in the low valuation region $[0, v_r)$ will neither buy nor rent the product; (b) Consumers in the intermediate region $[v_r, v_b^r)$ will rent the product from the platform; (c) Consumers in the high valuation region $[v_b^r, 1]$ will purchase the product. When $\frac{p_r}{\lambda} \geq p_b$, we can get $v_b^r < v_b < v_r$. In this scenario, as shown in Fig. 2, consumers with low valuation $[0, v_b)$ will neither purchase the product nor rent it and consumers with high valuation $[v_b, 1]$ will choose to purchase the product.

To address the more practical case where the retail and rental markets can coexist, we assume that $\frac{p_r}{\lambda} < p_b$.The demand of retail market and rental market are respectively given as:

$$D_b = 1 - \frac{p_b - p_r}{1-\lambda}, \quad D_r = \frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda} \tag{1}$$

3.2. The firm and the platform

We study two prevailing contract forms between the designer brand firm and rental platform, the wholesale contract and the agency contract, both of which are commonly used in the luxury fashion rental industry. For example, Rent the Runway, the dominant player in the luxury fashion rental economy, used to own the inventory on its website, purchasing from different fashion brands. Now it also allows partner designer brands to rent directly to consumers on the platform, making a profit by charging a commission fee. Currently, about 15% to 25% of the platform's inventory comes directly from outside fashion brands (Elizabeth, 2018a).

Under the wholesale contract, the designer brand firm sets a wholesale price w to the platform and a retail price p_b to consumers. Then the platform determines the rental price p_r. Under the agency contract, following Tan et al. (2016) and Geng et al. (2018), the platform acts as an agent and collectsα portion of rental revenue as a commission fee and the designer firm retains the $1-\alpha$ portion. Since α is usually governed by the industry standard, we assume that α is exogenous. The key difference from the wholesale contract is that the designer brand firm has the control to set the rental price p_r. According to business practice, the bulk of the revenue is collected by the firm, which leads us to focus on the case of $\alpha < 1/2$.

Without loss of generality, we assume the marginal cost of the product is zero. In line with the luxury fashion rental practice, consumers need to return the products at the end of the rental period. To get the closed-form solution, we do not consider the moral hazard of the renter and assume all consumers treat the product with great care and return the product on time. After one rental

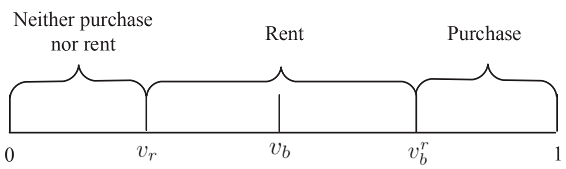


Fig. 1. Consumers Purchasing Choice when $\frac{p_r}{\lambda} < p_b$.

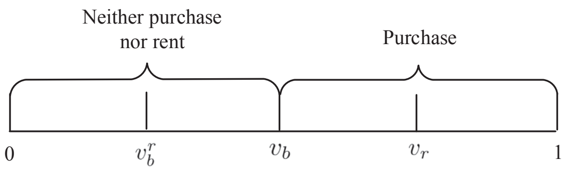


Fig. 2. Consumers Purchasing Choice when $\frac{p_r}{\lambda} \geq p_b$.

period, the returned products can be rented out again in the next period or sold to a discount store. We designate the salvage value of the returned product as ϵ and it is collected by the party who operates the rental business. That is, the salvage value is collected by the platform under the wholesale contract and designer brand firm under the agency contract.

4. Analysis

In this section, we consider a Stackelberg game to model the interaction between the firm and the platform and analyze the prevalent contracts associated with the luxury fashion rental industry. We assume that the upstream firm, as the market leader, makes the first move in the game. Then in the second stage, the platform, as the follower, responds to the firm's choice. We assume that both parties have full information about consumers' demand. We also analyze the single traditional retail channel and integrated channel model as the benchmarks for later comparisons. Fig. 3 illustrates the details of these models.

When there is no rental platform, the designer brand firm sells the product to consumers directly through a single traditional channel. In the integrated channel model, the firm and the platform no longer make individual decisions to maximize their own profits, but rather make collective decisions based on the overall profit of the supply chain. We assume α proportion of the rental revenue is collected by the platform. Table 2 summarizes the equilibrium outcomes of benchmarks and details are provided in Appendix A.

4.1. Wholesale model

Under a wholesale contract, the timing of events is as follows: (1) the firm determines the wholesale price w for the platform and the retail price p_b to consumers; (2) the platform determines the rental price p_r; (3) observing the retail price and rental price, consumers make their decisions. Utilizing the backward induction, we first consider the platform's problem:

$$\max_{p_r} \pi_P = (p_r - w)D_r + \epsilon D_r \tag{2}$$

After characterizing the optimal rental price, we substitute this solution back into the firm's problem, which is:

$$\max_{w, p_b} \pi_F = p_b D_b + w D_r \tag{3}$$

We summarize the equilibrium results under the wholesale contract in the following Lemma.

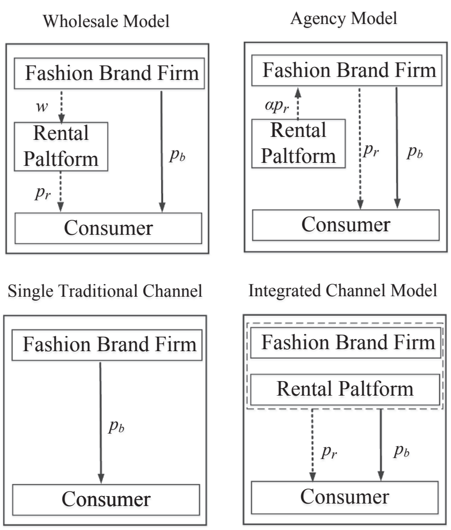


Fig. 3. Different Pricing Models.

Table 2
Equilibrium Outcomes of Benchmarks.

	Single Traditional Channel	Integrated Channel
Price		
Retail price, p_b	$\frac{1}{2}$	$\frac{1}{2}$
Rental price, p_r.	N/A	$\frac{\lambda-\epsilon}{2}$
Demand		
Retail market, D_b	$\frac{1}{2}$	$\frac{1-\lambda-\epsilon}{2(1-\lambda)}$
Rental market, D_r	N/A	$\frac{\epsilon}{2\lambda(1-\lambda)}$
Profits		
Firm's profit, π_F	$\frac{1}{4}$	$\frac{(1+\alpha)\epsilon^2-\alpha\lambda\epsilon+\lambda(1-\lambda)}{4\lambda(1-\lambda)}$
Platform's profit, π_P	N/A	$\frac{\alpha\epsilon(\lambda-\epsilon)}{4\lambda(1-\lambda)}$
Supply chain's profit, π_{SC}	$\frac{1}{4}$	$\frac{1}{4}+\frac{\epsilon^2}{4\lambda(1-\lambda)}$
Consumer surplus, CS	$\frac{1}{8}$	$\frac{1}{8}+\frac{\epsilon^2}{8\lambda(1-\lambda)}$
Social welfare, SW	$\frac{3}{8}$	$\frac{3}{8}+\frac{3\epsilon^2}{8\lambda(1-\lambda)}$

Lemma 1. *The optimal solutions under the wholesale contract are as follows:*

a) *Pricing:* $w=\frac{\lambda+\epsilon}{2}$, $p_b^W=\frac{1}{2}$, $p_r^W=\frac{2\lambda-\epsilon}{4}$.
b) *Demand:* $D_b^W=\frac{2(1-\lambda)-\epsilon}{4(1-\lambda)}$, $D_r^W=\frac{\epsilon}{4\lambda(1-\lambda)}$.
c) *Profits:* $\pi_F^W=\frac{1}{4}+\frac{\epsilon^2}{8\lambda(1-\lambda)}$, $\pi_P^W=\frac{\epsilon^2}{16\lambda(1-\lambda)}$, $\pi_{SC}^W=\frac{1}{4}+\frac{3\epsilon^2}{16\lambda(1-\lambda)}$.
d) *Consumer Surplus and Social Welfare:* $CS^W=\frac{1}{8}+\frac{\epsilon^2}{32\lambda(1-\lambda)}$, $SW^W=\frac{3}{8}+\frac{7\epsilon^2}{32\lambda(1-\lambda)}$.

It is straightforward to see from Lemma 1 that as the unit salvage value ϵ increases, the retail price p_b^W remains constant while the rental price p_r^W decreases. Consequently, the demand of the retail market shrinks (i.e., $\frac{\partial D_b^W}{\partial\epsilon}<0$), but the demand of the rental market increases (i.e., $\frac{\partial D_r^W}{\partial\epsilon}>0$). In equilibrium, we find that both the platform's profit and the firm's profit increase in ϵ. This result indicates that both the firm and the platform have an incentive to improve the unit salvage value of the returned product under the wholesale contract. The platform, for example, can improve the sophisticated cleaning services, while the firm can design more durable products.

As the discounted value of the rental product, λ, increases, both the wholesale price w and rental price p_r^W increase. For $\lambda\in(0,1/2]$, $\frac{\partial\pi_F^W}{\partial\lambda}<0$, $\frac{\partial\pi_P^W}{\partial\lambda}<0$, $\frac{\partial CS^W}{\partial\lambda}<0$; for $\lambda\in[1/2,1)$, $\frac{\partial\pi_F^W}{\partial\lambda}>0$, $\frac{\partial\pi_P^W}{\partial\lambda}>0$, $\frac{\partial CS^W}{\partial\lambda}>0$. As λ is associated with the length of the rental term, the implication from this result is that when the rental term is relatively low (i.e., $\lambda\in(0,1/2]$), the lower the rental term is, the more beneficial the firm and the platform will be. That is to say, both the firm and platform have an incentive to increase the difference between the valuation from purchasing the product and from renting the product. However, when λ is relatively high (i.e., $\lambda\in[1/2,1)$), both the firm and platform prefer a lower difference between these two markets. Next, we compare the equilibrium results under the wholesale contract with the single traditional channel and yield the following proposition.

Proposition 1. *The following results hold under the wholesale model and the single traditional channel:*

a) $p_b^W=p_b^N$.
b) $D_b^W<D_b^N$, $D_b^W+D_r^W>D_b^N$.
c) $\pi_F^W>\pi_F^N$, $\pi_{SC}^W>\pi_{SC}^N$.
d) $CS^W>CS^N$, $SW^W>SW^N$.

Our finding in Proposition 1 shows that the firm's profit under the wholesale contract with the presence of a rental platform outperforms that under the single traditional channel. The difference stems from the tradeoff between the market expansion effect and the cannibalization effect. When a rental platform exists, it competes with the designer brand firm, and some consumers convert from purchasing the product to renting it. With the same retail price (i.e., $p_b^W=p_b^N$), the retail market demand decreases (i.e., $D_b^W<D_b^N$), which we refer to as the *cannibalization effect.* At the same time, with the rental platform, some low-valuation consumers who initially do not purchase the product choose to rent it from the platform, resulting in aggregate market demand increases (i.e., $D_b^W+D_r^W>D_b^N$), which we refer to as the *market expansion effect.* Proposition 1 shows that, under the wholesale model, the expansion effect dominates the cannibalization effect. Consequently, the firm's profit is higher under the wholesale contract than under the single traditional channel. At the same time, under this business-to-consumer product sharing market, the consumer surplus and social welfare are also better than those under the traditional retail market.

Next, we compare the performance of the wholesale contract with another benchmark—the integrated channel model. The integrated channel model is known as the ideal supply chain scenario. Comparing the equilibrium outcomes under the wholesale model with those derived from the integrated channel model, we obtain the following proposition.

Proposition 2. *The following results hold under the wholesale model and the integrated channel:*

a) $p_b^W = p_b^I$, $p_r^W > p_r^I$.
b) $D_b^W > D_b^I$, $D_r^W < D_r^I$, $D_b^W + D_r^W < D_b^I + D_r^I$.
c) *For* $0 < \alpha < min\{\frac{\epsilon}{2(\lambda-\epsilon)}, \frac{1}{2}\}$, $\pi_F^W < \pi_F^I$, *otherwise* $\pi_F^W > \pi_F^I$;
For $0 < \alpha < min\{\frac{\epsilon}{4(\lambda-\epsilon)}, \frac{1}{2}\}$, $\pi_P^W > \pi_P^I$, *otherwise* $\pi_P^W < \pi_P^I$;
$\pi_{SC}^W < \pi_{SC}^I$.
d) $CS^W < CS^I$, $SW^W < SW^I$.

These results show that when $\lambda > 2\epsilon$ (i.e., $\frac{\epsilon}{2(\lambda-\epsilon)} < \frac{1}{2}$), if $\alpha \in (0, \frac{\epsilon}{4(\lambda-\epsilon)}]$, then the firm favors the integrated channel model while the platform favors the wholesale model; if $\alpha \in (\frac{\epsilon}{4(\lambda-\epsilon)}, \frac{\epsilon}{2(\lambda-\epsilon)}]$, then both the rental platform and the firm prefer the integrated channel model; if $\alpha \in (\frac{\epsilon}{2(\lambda-\epsilon)}, \frac{1}{2})$, then the firm prefers the wholesale model while the platform prefers the integrated channel model. This implies that when the revenue-sharing proportion α belongs to an intermediate range, both the firm and the platform prefer the integrated channel model. Similarly, when $\frac{2}{3}\epsilon < \lambda < 2\epsilon$ (i.e., $\frac{\epsilon}{4(\lambda-\epsilon)} < \frac{1}{2} < \frac{\epsilon}{2(\lambda-\epsilon)}$), if $\alpha \in (\frac{\epsilon}{4(\lambda-\epsilon)}, \frac{1}{2}]$, then both the rental platform and the firm prefer the integrated channel model. This is because compared with the wholesale model, the overall profit of the supply chain increases under the integrated channel model, and when the rental market revenue is appropriately distributed, both parties are satisfied with an increased profit. However, when $\lambda < \frac{2}{3}\epsilon$ (i.e., $\frac{1}{2} < \frac{\epsilon}{4(\lambda-\epsilon)}$), the firm always prefers the integrated channel model while the platform prefers the wholesale model. Consistent with the expectation, the consumer surplus and social welfare under the integrated channel model are also better than those under the wholesale model.

4.2. Agency model

Under an agency contract, the timing of events is as follows: (1) the firm determines the rental price p_r and retail price p_b simultaneously; (2) after observing the retail price and rental price, consumers make their choices. In this scenario, the profits of the firm and the platform are as follows:

$$\pi_F = p_b D_b + (1-\alpha) p_r D_r + \epsilon D_r \tag{4}$$

$$\pi_P = \alpha p_r D_r \tag{5}$$

Lemma 2 summarizes the equilibrium outcomes under the agency contract. For convenience, we define $m = 4(1-\alpha) - (2-\alpha)^2\lambda$, $n = 2(1-\lambda) + \alpha\lambda$, and $l = 4(1-2\alpha)(1-2\lambda)$.

Lemma 2. *Under the condition* $4(1-\alpha) - (2-\alpha)^2\lambda > 0$ *and* $\alpha < \frac{2\epsilon}{\lambda}$, *the optimal solutions under the agency contract are as follows:*

a) *Pricing:* $p_b^A = \frac{2(1-\alpha)(1-\lambda)-\alpha\epsilon}{m}$, $p_r^A = \frac{\lambda(2-\alpha)(1-\lambda+\epsilon)-2\epsilon}{m}$.
b) *Demand:* $D_b^A = \frac{2(1-\lambda-\epsilon)+\alpha(\epsilon-2+\lambda(3-\alpha))}{m}$, $D_r^A = \frac{2\epsilon-\alpha\lambda}{\lambda m}$.
c) *Profits:*

$$\pi_F^A = \frac{\epsilon^2 - \alpha\lambda\epsilon + (1-\alpha)\lambda(1-\lambda)}{\lambda m}, \quad \pi_P^A = \frac{\alpha(2\epsilon - \alpha\lambda)(\lambda(2-\alpha)(1-\lambda+\epsilon) - 2\epsilon)}{\lambda m^2}$$

$$\pi_{SC}^A = \frac{\epsilon^2(4(1-2\alpha)(1-\lambda) - 3\alpha^2\lambda) + 2\alpha^2\lambda\epsilon n + \lambda(1-\lambda)(4(1-\alpha)^2 + (\alpha-2)(\alpha(2\alpha-3)+2)\lambda)}{\lambda m^2}$$

d) *Consumer Surplus and Social Welfare*:

$$CS^A = \frac{\epsilon^2(4(1-\lambda) + \alpha^2\lambda) - 2\alpha^2\lambda\epsilon n + \lambda(l + \alpha^2(4 + (4\alpha - 11)\lambda) + (1-\alpha)(2-\alpha)(2-(1-\alpha)\alpha)\lambda^2)}{2\lambda m^2}$$

$$SW^A = \frac{\epsilon^2(4(3-4\alpha)(1-\lambda) - 5\alpha^2\lambda) + 2\alpha^2\lambda\epsilon n + \lambda(3l + \alpha^2(12 + (8\alpha - 33)\lambda) + (2-\alpha)(6-(3-\alpha)^2\alpha)\lambda^2)}{2\lambda m^2}$$

By comparing Lemma 1 and Lemma 2, we find that under the agency contract, the retail price p_b^A is no longer a constant but varies with parameters, which is different from that under the wholesale contract. In addition, we can show that p_b^A is strictly less than $\frac{1}{2}$(the retail price under wholesale contact). For the consumers with high valuation who purchase the product, their consumer surplus also become higher. This indicates that under the agency contract, the consumer who buys the product can benefit more from the presence of the rental platform. We next compare the equilibrium under the agency contract with the benchmark of a single traditional channel.

Proposition 3. *The following results hold under the agency model and the single traditional channel:*

a) $p_b^A < p_b^N$.
b) $D_b^A < D_b^N$, $D_b^A + D_r^A > D_b^N$.
c) $\pi_F^A > \pi_F^N$, $\pi_{SC}^A > \pi_{SC}^N$.
d) $CS^A > CS^N$, $SW^A > SW^N$

Similar to the wholesale contract, the firm's profit under the agency contract outperforms that under a single traditional retail channel. Even with a lower retail price (i.e., $p_b^A < p_b^N$), the *cannibalization effect* still leads to lower demand in the retail market (i.e., $D_b^A < D_b^N$). At the same time, the *expansion effect* makes the aggregate demand of the retail market and rental market higher than without a rental market (i.e., $D_b^A + D_r^A > D_b^N$). Proposition 3 indicates that the profit benefit from the expansion effect dominates the profit loss incurred by competing. Consequently, the firm's profit is higher under the agency model. At the same time, under the agency contract, the consumer surplus and social welfare are also better than those under the traditional retail market. Proposition 1 and Proposition 3 explain why fashion retailers are willing to work with rental platforms in practice.

4.3. Comparison

To further explore the mechanisms of the wholesale contract and the agency contract in the fashion rental industry, in this section, we compare profits of the firm and platform under these two contracts respectively and analyze the critical factors that determine the firm's and the platform's choices between these two contracts. We summarize the firm's contract choice in the following proposition. For convenience, we define $g(\epsilon) = (1-2\lambda)\epsilon^2 - 4(1-\lambda)\lambda\epsilon + 2(1-\lambda)\lambda^2$, $\alpha_1 = \frac{2(1-\lambda)\epsilon}{(1-\lambda)(2\lambda-\epsilon)+\sqrt{(1-\lambda)g(\epsilon)}}$, and ϵ^* satisfies $g(\epsilon^*) = 0$.

Proposition 4. *By comparing the firm's profits under different contracts, we can get:*

a) *If* $\epsilon \geq \epsilon^*$, *then* $\pi_F^A > \pi_F^W$.
b) *If* $\epsilon < \epsilon^*$ *and* $\alpha_1 \geq \frac{1}{2}$, *then* $\pi_F^A > \pi_F^W$ *for* $\alpha \in (0, \frac{1}{2})$.
c) *If* $\epsilon < \epsilon^*$ *and* $\alpha_1 < \frac{1}{2}$, *then* $\pi_F^A > \pi_F^W$ *for* $\alpha \in (0, \alpha_1)$ *and* $\pi_F^A < \pi_F^W$ *for* $\alpha \in (\alpha_1, \frac{1}{2})$.

Proposition 4 reveals that the firm's choice between the agency contract and the wholesale contract mainly relies on the revenue-sharing proportion α and the salvage value ϵ. The result of part a) shows that when the salvage value is relatively large, the firm always prefers the agency contract no matter how the proportion is valued. This is due to the fact that under the agency contract, the salvage value of the returned products is collected by the firm, whereas under the wholesale contract it is collected by the platform. A higher ϵ suggests that the firm can benefit more, which encourages the firm to choose the agency contract. When ϵ is relatively small, the firm's contract choice depends mainly on the revenue-sharing proportion α. Common wisdom suggests that the firm is willing to choose the agency contract when α is low and the wholesale contract when α is high. The result of part c) is in line with this expectation.

Next, we analyze the platform's contract choice. For convenience, we define $\epsilon_1 = \frac{4\alpha\lambda(1-\lambda)(2(4(1-\lambda)+\alpha^2\lambda)-m\sqrt{\alpha+2})}{m^2+32\alpha(1-\lambda)n}$, $\epsilon_2 = \frac{4\alpha\lambda(1-\lambda)(2(4(1-\lambda)+\alpha^2\lambda)+m\sqrt{\alpha+2})}{m^2+32\alpha(1-\lambda)n}$ and $\bar{\epsilon} = min\{\frac{(2-\alpha)\lambda(1-\lambda)}{n}, \frac{(3-\alpha)\alpha\lambda+2(1-\alpha-\lambda)}{2-\alpha}\}$, where $\bar{\epsilon}$ is the upper critical value of ϵ.

Proposition 5. *By comparing the platform's profits under different contracts, we can get:*

a) *If* $\epsilon_1 < \bar{\epsilon} \leq \epsilon_2$, *then* $\pi_P^A < \pi_P^W$ *for* $\epsilon \in (\frac{\alpha\lambda}{2}, \epsilon_1)$, *and* $\pi_P^A > \pi_P^W$ *for* $\epsilon \in (\epsilon_1, \bar{\epsilon})$.
b) *If* $\bar{\epsilon} > \epsilon_2$, *then* $\pi_P^A < \pi_P^W$ *for* $\epsilon \in (\frac{\alpha\lambda}{2}, \epsilon_1)$ *and* $\epsilon \in (\epsilon_2, \bar{\epsilon})$, *and* $\pi_P^A > \pi_P^W$ *for* $\epsilon \in (\epsilon_1, \epsilon_2)$.

Proposition 5 reveals that the platform's contract choice depends mainly on the salvage value ϵ, and the platform is willing to choose the agency contract only when the salvage value ϵ falls into a certain range. When ϵ is relatively large (i.e., $\epsilon \in (\epsilon_2, \bar{\epsilon})$), the platform prefers the wholesale contract. This is because a higher ϵ encourages the platform to choose the wholesale contract to collect the salvage value of the returned products. Interestingly, we find that when ϵ falls into a small range (i.e., $\epsilon \in (\frac{\alpha\lambda}{2}, \epsilon_1)$), the platform also favors the wholesale model. This is because under the agency contract, as ϵ decreases, both the retail price and the rental price increase, and the rental market demand decreases while the retail market increases (i.e., $\frac{\partial p_b^A}{\partial \epsilon} < 0$, $\frac{\partial p_r^A}{\partial \epsilon} < 0$, $\frac{\partial D_b^A}{\partial \epsilon} < 0$, $\frac{\partial D_r^A}{\partial \epsilon} > 0$). This implies that when ϵ is relatively small, the firm will aggressively increase the rental price to increase the demand of the retail market, which induces the profit loss of the platform. As a result, the platform is willing to choose the wholesale contract.

Fig. 4 illustrates the findings of Proposition 4 and Proposition 5. It is easy to find that there exists an area for the salvage value ϵ and the revenue-sharing proportion α in which the agency contract will benefit both the firm and the platform. In the area highlighted blue, the profits of the firm, the platform, and the supply chain are all improved compared with the wholesale contract. This Pareto improving region provides practical guidance for platform operation in the fashion rental industry.

5. Extensions

To check the robustness of our main findings, in this section, we analyze several alternative scenarios. First, we consider the

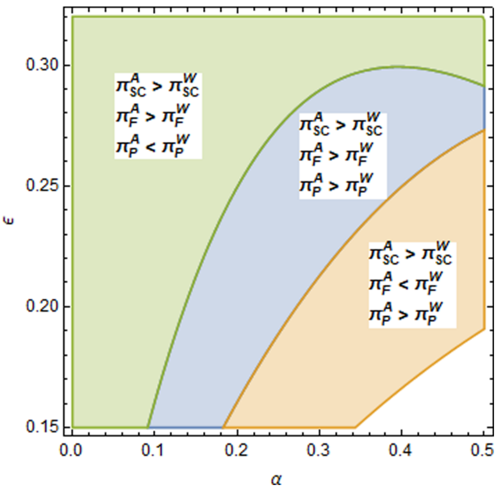


Fig. 4. Illustration of Firm and Platform's Contract Choices ($\lambda = 0.6$).

existence of different consumer segments, where some consumers always prefer to rent the product rather than purchase it. Second, we consider consumers' conspicuous behavior for the retail market. Next, we consider the competition between rental platforms. Finally, we extend the base model to a two-period model setting. These extensions illustrate the robustness of our main findings to alternative model settings and also provide new insights on the impact of the business-to-consumer luxury fashion sharing.

5.1. Different consumer segments

We first consider the general model by relaxing the assumptions on consumers. In the base model, we assume that consumers always prefer purchasing the products when the utility from buying the product is higher than that from renting it. However, we observe that some consumers prefer to rent the product rather than purchase it even when they have a high valuation for the product. For example, many consumers choose to rent an evening gown or tuxedo instead of purchasing one because they can only be worn in a formal setting. As a result, although consumers have a quite high valuation on formal clothing, they prefer the rental service when it is available. We call this segment of consumers as *rental enthusiastic consumers*. If there is no rental market, they have to purchase the products, but otherwise, they always prefer rental to purchase.

We assume that θ proportion of consumers are regular consumers that behave the same as in the base model, and $1-\theta$ proportion of consumers have a strong preference for the rental service, where $0 < \theta < 1$. We use superscripts "*Wr*" and "*Ar*" to represent the wholesale model and the agency model, respectively. We first compare the designer brand firm's profit under the wholesale model with the single traditional channel benchmark. For convenience, we define:

$$\theta_{W1}^r \equiv \frac{(1-\lambda)\lambda(\epsilon\,(1-2\lambda) - \epsilon^2 - (1-\lambda)^2) - \sqrt{(2\,\epsilon^2 + 2\,\epsilon\,(-1+\lambda) + (-1+\lambda)^2)(-1+\lambda)^2\lambda^2}}{\lambda^2(1-\epsilon-\lambda)^2}$$ and

$$\theta_{W2}^r \equiv \frac{\sqrt{(2\,\epsilon^2 + 2\,\epsilon\,(-1+\lambda) + (-1+\lambda)^2)(-1+\lambda)^2\lambda^2} - (1-\lambda)\lambda(\epsilon^2 + (1-\lambda)^2 - \epsilon\,(1-2\lambda))}{\lambda^2(1-\epsilon-\lambda)^2}$$

Proposition 6. *Comparing the firm' profit under the wholesale model with that derived from the single traditional channel, we can get:*

a) *If* $\theta_{W2}^r > 0$, *when* $\min\{\theta_{W1}^r, 0\} < \theta < \theta_{W2}^r$, *then* $\pi_F^{Wr} < \pi_F^N$, *the firm benefits more from the single traditional channel than the wholesale model.*
b) *If* $\theta_{W2}^r > 0$, *when* $0 < \theta \le \min\{\theta_{W1}^r, 0\}$ *or* $\theta_{W2}^r \le \theta \le 1$, *then* $\pi_F^{Wr} > \pi_F^N$, *the firm benefits more from the wholesale model than the single traditional channel.*
c) *If* $\theta_{W2}^r < 0$, *then* $\pi_F^{Wr} > \pi_F^N$, *the firm benefits more from the wholesale model than the single traditional channel.*

Proposition 6 presents an interesting new insight: Whenθ satisfies certain conditions, the firm may earn a lower profit with the presence of a rental platform than in a single traditional retail channel. Intuitively, the rental market will expand the demand because of the *expansion effect*. However, as high valuation consumers transfer to the rental market, the firm will suffer a loss in the retail market. When the loss in the retail market exceeds the extra rental market profit, like the low salvage value of the returned product or low discounted value due to product returns, the *expansion effect* cannot dominate the cannibalization effect. Consequently, the proportion of rental enthusiastic consumers determines whether the presence of a rental platform is beneficial to the designer brand firm.

Next, we compare the equilibrium solutions under the agency contract with the single traditional channel benchmark. For convenience, we define

$$\theta_A^r = \frac{4(-1+\lambda)(\epsilon^2 - \lambda + \alpha\lambda + 2\epsilon\lambda - 2\alpha\epsilon\lambda + \lambda^2 - 2\alpha\lambda^2 + \alpha^2\lambda^2)}{\lambda(4 - 4\alpha - 8\epsilon + 4\alpha\epsilon + 4\epsilon^2 - 8\lambda + 12\alpha\lambda - 3\alpha^2\lambda + 8\epsilon\lambda - 8\alpha\epsilon\lambda + 4\lambda^2 - 8\alpha\lambda^2 + 4\alpha^2\lambda^2)}$$

Proposition 7. *Comparing the firm' profit under the agency model with that derived from the single traditional channel, we can get:*

a) *If* $0 < \theta_A^r < 1$, $0 < \theta < \theta_A^r$, *then* $\pi_F^{Ar} < \pi_F^N$, *the firm benefits more from the single traditional channel than the agency model.*
b) *If* $0 < \theta_A^r < 1$, $\theta_A^r < \theta \leq 1$, *then* $\pi_F^{Ar} > \pi_F^N$, *the firm benefits more from the agency model than the single traditional channel.*

Similar to Proposition 6, Proposition 7 reveals that the firm may not be better off under the agency contract when the proportion of rental enthusiastic consumers is high. This result is driven by the comparison between the benefit in demand and loss in high valuation consumers. When the proportion of rental enthusiastic consumers is large, although the rental market increases the aggregate demand, it still cannot make up the loss of profit caused by the high valuation consumers in the retail market transferring to the rental market. That is to say, the weak *expansion effect* of the rental market cannot dominate the cannibalization effect. Consequently, the firm prefers the single traditional channel.

To summarize, when considering rental enthusiastic consumers, the results are consistent with our core insight in the base model: When the proportion of rental enthusiastic consumers is small, the presence of a rental platform benefits the designer brand firm. This confirms the robustness of the base model once more. Moreover, the existence of rental enthusiastic consumers also provides a new interesting insight: The high valuation consumers' actions affect the firm's strategy about the rental market. Firms prefer the market with a rental platform if most of the high valuation consumers prefer to purchase products.

5.2. Consumers' conspicuous behavior

In the luxury fashion market, conspicuousness is an important factor that affects customers' perceived value. Consumers often choose to purchase designer apparel, jewelry, watches, and other products that are "hard to get" (i.e., unique) to conspicuously display their social status (Amaldoss and Jain, 2005a; Tereyagoglu and Veeraraghavan, 2012; Amaldoss and Jain, 2015). Grilo et al. (2001) describe conspicuous consumption as a negative externality that is caused by the increase in consumers purchasing the same product. In this subsection, we incorporate consumers' conspicuous behavior into the base model and investigate the impact of the conspicuous behavior on different parties' channel decisions.

We assume that consumers who purchase the luxury product desire uniqueness. Therefore, as the number of people using the product increases, consumers' utility from buying the product decreases. Following Amaldoss and Jain (2005b), consumers' utility from purchasing the product is $u_b = v - p_b - \delta(D_b + D_r)$, where δ captures consumers' desire for uniqueness. For the consumers who rent the product, they pay little attention to uniqueness, and therefore the utility from renting the product is $u_r = \lambda v - p_r$. To check the robustness of core insights in the base model, we focus on comparing profits under different models. First, we compare the profit of the designer brand firm with and without a rental platform.

Proposition 8. *When considering conspicuous behavior, the firm's profits under the wholesale contract and the agency contract are always higher than under the single traditional channel.*

In the past, it was a common practice for marketers to promote conspicuous products by emphasizing the uniqueness of their products. Some luxury companies use exclusive channels to sell their products because they fear that wide availability may damage the company's reputation. When deciding whether to cooperate with rental platforms, this has become a principal consideration for many luxury firms. Our results suggest that, considering the consumers' conspicuous behavior, the profit benefit from the expansion effect of the rental market always dominates the profit loss from the cannibalization effect incurred by competing and consumers' conspicuous behavior. As a result, the firm will get a higher profit with a rental platform than without it. This result reinforces our findings in the base model. We find that the rental platform is actually beneficial to the upstream designer brand even with the existence of conspicuous behavior.

Next, we analyze the contract choice of the designer brand firm and the rental platform under the wholesale contract and the agency contract. Although we can obtain a closed-form solution, the comparison of profits is fairly complex. Therefore, we investigate

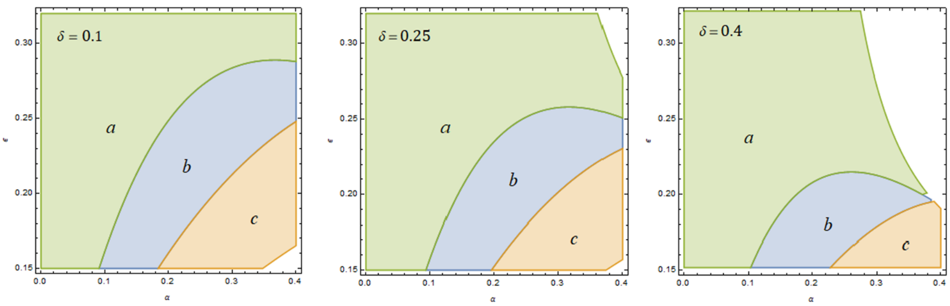


Fig. 5. Illustration of Firm and Platform's Contract Choices Considering Conspicuous Behavior.

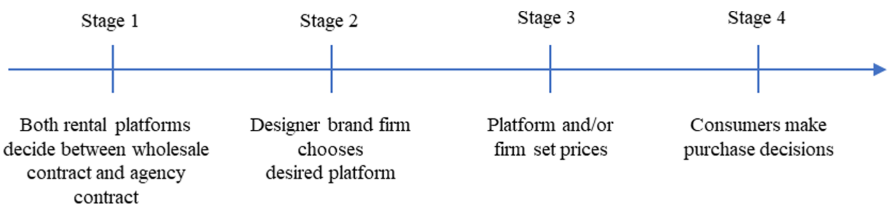


Fig. 6. Timeline of Model Considering Platform Competition.

the effect of conspicuous behavior through numerical experiments and provide several representative examples in Fig. 5 with different values of δ.

In general, we observe that when considering consumers' conspicuous behavior, the results are consistent with our main insights in the base model. That is, the choice between the agency contract and the wholesale contract mainly relies on the portion of revenue-sharing α and the salvage valueϵ. Whenα is very small or ϵ is very large (i.e., Area a in Fig. 5), the firm is willing to choose the agency contract while the platform would like to choose the wholesale contract (i.e., $\pi_F^A > \pi_F^W$, $\pi_P^A < \pi_P^W$); when α is very large and ϵ is very small (i.e., Area c), the firm favors the wholesale contract while the platform is more willing to choose the agency contract (i.e., $\pi_F^A < \pi_F^W$, $\pi_P^A > \pi_P^W$). In these two scenarios, although the overall profit of the supply chain is higher under the agency contract, both parties cannot reach coordination. When α is relatively large and ϵ is relatively small (i.e., Area b), there exists a region of α and ϵ in which both the firm and the platform would like to choose the agency contract, and the agency contract can improve the supply chain performance.

Furthermore, we find that as δ increases, the region of coordination (Area b) decreases. The intuition is that when consumers' utility from purchasing the product is affected by the demand of the total market, the firm has a stronger incentive to acquire the pricing power. That is, the firm prefers the agency contract on a larger region compared with the base model. At the same time, the platform prefers the wholesale contract in a larger region, as more consumers will rent the product instead of purchasing it compared with the base model. Consequently, the area in which both the firm and the platform are willing to choose the agency contract decreases.

5.3. Platform competition

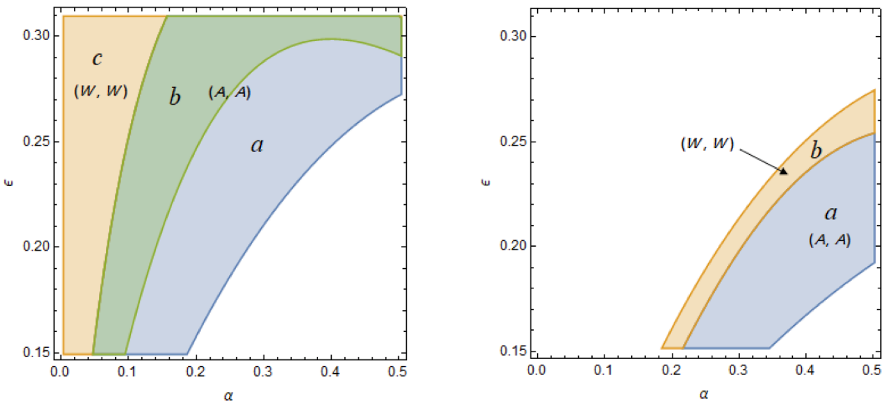
With the booming of sharing economy, more and more platforms enter the luxury fashion rental market and in some cases, different rental platforms will compete for a single designer brand firm's rental business (Kate, 2019). In this subsection, we analyze an alternative model with two competing rental platforms and one designer brand firm, and examine the impact of platform competition. Fig. 6 illustrates the timeline of this alternative setting.

In stage 1, given the contract choices of the rental platforms, denoted by P1 and P2, there are four possible scenarios: both rental platforms choose agency contracts (AA), P1 chooses the agency contract and P2 chooses the wholesale contract (AW), P1 chooses the wholesale contract and P2 chooses the agency contract (WA), and both platforms adopt wholesale contracts (WW). The following proposition summarizes the equilibrium outcomes with two competing rental platforms.

Proposition 9. *When considering the competition between two rental platforms, both firms will choose the same form of contract, and the equilibrium result is either AA or WW.*

To understand the intuition behind Proposition 9, we suppose that rental platforms choose different contracts in stage 1 and assume P1 chooses the wholesale contract and P2 adopts the agency contract. Then in stage 2, if the designer brand firm chooses to work with P1, it immediately follows the subsequent interaction in Section 4.1, and from Lemma 1, the profit for the firm is $\pi_F^W = \frac{1}{4} + \frac{\epsilon^2}{8\lambda(1-\lambda)}$. If the designer brand firm chooses to cooperate with P2, it follows the subsequent interaction in Section 4.2, and from Lemma 2, the profit for the firm is $\pi_F^A = \frac{\epsilon^2 - \alpha\lambda\epsilon + (1-\alpha)\lambda(1-\lambda)}{\lambda m}$. Following Proposition 4, the firm always chooses the desired platform with a higher profit. For example, when $\epsilon \geq \epsilon^*$, the firm has a higher profit under the agency contract and will choose to work with platform P2. Therefore, the profit of P2 is $\pi_P^A = \frac{\alpha(2\epsilon - \alpha\lambda)(\lambda(2-\alpha)(1-\lambda+\epsilon) - 2\epsilon)}{\lambda m^2}$, and the profit of P1 is zero. However, if P1 also chooses the agency contract, then the firm will randomly pick a platform, and each platform can get an expected payoff $\pi_P^A/2$. Therefore, P1 will choose the same contract as P2.

To further illustrate the impact of competition on the platforms' contract choice, we conduct numerical experiments used in the base model and show the difference of platforms' choices. As shown in Fig. 7, we observe that when $\pi_F^A > \pi_F^W$, there exists a region (Area b in a)) in which both rental platforms choose agency contracts, although in this case, the wholesale contract for the platform is more profitable than the agency contract (i.e., $\pi_P^A < \pi_P^W$). The intuition is that, in this scenario, if the competitor chooses the wholesale contract, the platform that adopts the agency contract will dominate the competitor because $\pi_F^A > \pi_F^W$ and it also can get a higher profit from choosing the agency contract because $\pi_P^A > \pi_P^W/2$. Similarly, when $\pi_F^A < \pi_F^W$, there also exists a region (Area b in b)) in which both rental platforms choose wholesale contracts, although in this case $\pi_P^A > \pi_P^W$. This indicates that the competition between platforms can change the platform's contract preference in some cases.



a) $\pi_F^A > \pi_F^W$ b) $\pi_F^A < \pi_F^W$

Fig. 7. Illustration of Competing Platforms' Contract Choices.

5.4. Two-period rental model

For exposition, we focus on the one-period setting in the base model. However, one can argue that product can usually be rented for multiple times in practice. In this subsection, we extend the single-period setting to a two-period model. For some luxury fashion products, especially like designer fashion clothes, they usually have a limited life cycle (Caniato et al., 2012). We normalize the life cycle of the product to be 1. Note that the luxury fashion renting is usually a short-term event, therefore we assume the discounted value λ is less than 0.5. In the first rental period, similar to the base model, consumers' valuation from buying the product is v, and the valuation of the rental product is λv. Then in the second rental period, the product's expected usage time will be reduced to $(1-\lambda)$. That is, in the second rental period, consumers' valuation of buying the product is reduced to $(1-\lambda)v$, and renting valuation is still λv. To reduce the complexity of analysis, we assume that the rental platform and the designer brand firm will strategically decide prices but not dynamically adjust prices in different periods.

Following the base model, in the first period, the demands of retail market and rental market are $D_{1b} = 1 - \frac{p_b - p_r}{1-\lambda}$ and $D_{1r} = \frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}$, respectively. In the second period, consumers' utility from purchasing and renting are $(1-\lambda)v - p_b$ and $\lambda v - p_r$, respectively. They would strategically choose among purchasing, renting, and neither purchasing nor renting. The demand can be expressed as:

$$D_{2b} = \begin{cases} 1 - \frac{p_b - p_r}{1-2\lambda}, & \frac{p_b - p_r}{1-2\lambda} < 1 \\ 0, & \frac{p_b - p_r}{1-2\lambda} > 1. \end{cases}, \quad D_{2r} = \begin{cases} \frac{p_b - p_r}{1-2\lambda} - \frac{p_r}{\lambda}, & \frac{p_b - p_r}{1-2\lambda} < 1 \\ 1 - \frac{p_r}{\lambda}, & \frac{p_b - p_r}{1-2\lambda} > 1. \end{cases} \tag{6}$$

To examine the robustness of our key findings, we first compare the profit of the designer brand firm under the wholesale contract with the single traditional channel.

Proposition 10.. *In the two-period rental model, the designer brand firm's profit under the wholesale contract is always higher than that under the single traditional channel.*

Compared with the demand in the first period, we can easily find that the demand of the retail market in the second period decreases (i.e., $D_{2b} < D_{1b}$), while the demand of the rental market in the second period increases significantly (i.e., $D_{2r} > D_{1r}$). This implies that when considering multi-periods rental, the rental market will further cannibalize the retail market. Proposition 10 shows that, under the wholesale model, the gain from the expansion effect of the rental market still dominates the loss from the cannibalization effect. This result further reinforces our main findings in the base model.

Next, we compare the equilibrium characteristics under different models. As shown in Equation (6), based on pricing decisions made by the designer brand firm and/or the rental platform, there are two cases of the second period's demand: (a) The rental market partially cannibalizes the retail market with a relatively small difference between retail prices and rental prices (i.e., $\frac{p_b - p_r}{1-2\lambda} < 1$). We call this strategy as the *low price difference strategy*; (b) The rental market totally cannibalizes the retail market with a relatively large difference between retail prices and rental prices (i.e., $\frac{p_b - p_r}{1-2\lambda} > 1$). We call this strategy as the *high price difference strategy*. Comparing the equilibrium outcomes under different contracts, we obtain the following proposition.

Proposition 11. *Under the wholesale model, the designer brand firm always chooses the low price difference strategy. However, under the agency model, the designer brand firm will strategically choose between low price difference strategy and high price difference strategy.*

The intuition behind Proposition 11 is that, when one product can be rented multiple times, the order quantity of the platform to the designer brand firm is the maximum demand of two periods' rental markets. That is to say, if the designer brand firm chooses the wholesale contract, although the rental demand of the second period increases, the firm can only partially benefit from the increased

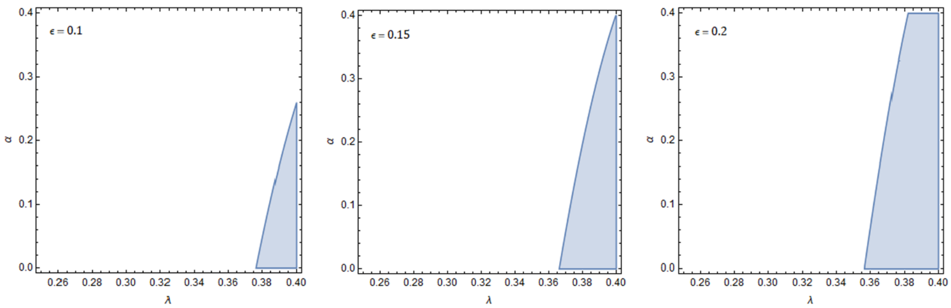


Fig. 8. Illustration of Firm's Strategy Preference under Agency Contract.

demand. In this case, the designer brand firm has no incentive to give up the retail market, so she always chooses the low price difference strategy to compete with the rental platform. However, the designer brand firm has the power to set the rental price under the agency contract, so she has an incentive to strategically adjust pricing strategy to get more profit. We numerically compare the firm's profit under different strategies. For ease of discussion, we denote the firm's profit under low price difference strategy as π_F^{AL} and profit under high price difference strategy as π_F^{AH}. As shown in Fig. 8, as ϵ increases, λ increases and α decreases, the region in which the firm prefers the high price difference strategy will increase. The intuition is that, as ϵ increases, λ increases and α decreases, the designer brand firm benefits more from the rental market, which encourages the firm to give up the second period's retail market and choose the high price difference strategy.

6. Conclusions

The luxury fashion rental business has become a major trend with the rapid growth of the sharing economy. Even designer brand firms and traditional retailers are trying to find ways to cooperate with these platforms. In this paper, we analytically study the impact of rental online marketplace on luxury fashion brands. Specifically, there exists a designer brand firm that sells a luxury fashion product and a rental platform that can either rent the purchased product from the brand firm or allow the firm to use its platform to rent out directly to consumers. We analyze the contract choice between the firm and the platform. Our analysis reveals several important findings.

To begin with, the presence of a rental platform can benefit the designer brand firm. This is due to the tradeoff between the market expansion effect and the cannibalization effect of the rental market. The rental platform provides consumers with an alternative way of owning luxury fashion goods, thereby reducing the demand in the retail market (the cannibalization effect). At the same time, for some consumers with low-valuation, it turns out that they initially don't buy the product, but now choose to rent it from the platform, so that the aggregate market demand increases (the expansion effect). Our analysis shows that under both the agency and wholesale contracts, the expansion effect dominates the cannibalization effect, and the designer brand firm benefits more with a rental market than without. This result provides a possible explanation for why many fashion retailers cooperate with rental platforms.

Further, we find that the revenue-sharing proportion and the salvage value have a crucial impact on the optimal contract decision of the firm and platform. We show that when the proportion of revenue need to share is small and the salvage value is relatively large, the firm favors the agency contract, while the platform would like to choose the agency contract when the salvage value falls into a certain range. Our results also reveal that when the portion of revenue-sharing is relatively large and salvage value is relatively small, both the firm and the platform benefit more under the agency contract than under the wholesale contract. In this scenario, the agency contract can improve the overall profit of the supply chain.

Third, compared with the wholesale model, the single traditional channel model, and the integrated channel model, the designer brand firm will charge a lower retail price under the agency model. This indicates that when the firm has the power to set the rental price of the new channel, the equilibrium price of the traditional retail channel will decrease. For high valuation consumers who purchase the product, under the agency contract, their consumer surplus will increase and benefit more from the presence of a rental platform.

We briefly point out some caveats of this study which may provide interesting directions for future research. First, this study only considers two prevalent contract forms between the designer brand firm and the rental platform. Future research can explore other supply chain contracts and cooperation modes. For example, some platforms help the fashion retailer to launch its own rental services (Elizabeth, 2018b). Furthermore, this research has assumed that both the firm and the platform have full information about the demand. Studying the effects of information asymmetry on the rental market would also be an interesting direction. Notwithstanding these limitations, this study takes the first step to understand the impact of a rental market on luxury fashion brands and contributes to the emerging field of research on sharing economy.

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Appendix A

Proof of single traditional channel

Given the demand function in Section 3, i.e., $D_b^N = 1 - p_b$, the designer brand firm's optimal problem becomes: $\max_{p_b} \pi_F = p_b(1 - p_b)$. Setting the first derivative of p_b equal to zero, we get $p_b^{N*} = \frac{1}{2}$. Substituting the optimal retail price p_b^{N*} into demand and profit functions, we have:

$$D_b^{N*} = 1 - p_b^{N*} = \frac{1}{2}$$

$$\pi_F^{N*} = p_b^{N*}(1 - p_b^{N*}) = \frac{1}{4}$$

Consumer surplus CS^N and Social welfare SW^N can be calculated as follows:

$$CS^N = \int_{\frac{1}{2}}^{1} (v - \frac{1}{2})dv = \frac{1}{8}$$

$$SW^N = \pi_F^{N*} + CS^N = \frac{3}{8}$$

Proof of the integrated channel model

Given the demand function $D_b = 1 - \frac{p_b - p_r}{1-\lambda}$, $D_r = \frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}$, the supply chain faces the following optimization problem,

$$\max_{p_b, p_r} \pi_{SC} = p_b(1 - \frac{p_b - p_r}{1 - \lambda}) + p_r(\frac{p_b - p_r}{1 - \lambda} - \frac{p_r}{\lambda}) + \epsilon(\frac{p_b - p_r}{1 - \lambda} - \frac{p_r}{\lambda})$$

We first show that π_{SC} is jointly concave in p_b and p_r by examining the Hessian.

$$H = \begin{bmatrix} \frac{-2}{1-\lambda} & \frac{2}{1-\lambda} \\ \frac{2}{1-\lambda} & \frac{-2}{(1-\lambda)\lambda} \end{bmatrix}$$

It is easy to find that the Hessian matrix is negative definite. Setting the first derivative of p_b and p_r equal to zero, we get $p_b^{I*} = \frac{1}{2}$ and $p_r^{I*} = \frac{\lambda - \epsilon}{2}$. It is easy to check that the results satisfy the consumption $\frac{p_r}{\lambda} < p_b$. Consequently, we have the following equilibrium results:

$$D_b^{I*} = 1 - \frac{p_b - p_r}{1 - \lambda} = \frac{1 - \lambda - \epsilon}{2(1 - \lambda)}, \; D_r^{I*} = \frac{p_b - p_r}{1 - \lambda} - \frac{p_r}{\lambda} = \frac{\epsilon}{2\lambda(1 - \lambda)}$$

$$\pi_F^{I*} = p_b D_b + (1 - \alpha)p_r D_r + \epsilon D_r = \frac{(1 + \alpha)\epsilon^2 - \alpha\lambda\epsilon + \lambda(1 - \lambda)}{4\lambda(1 - \lambda)}, \; \pi_P^{I*} = \alpha p_r D_r = \frac{\alpha\epsilon(\lambda - \epsilon)}{4\lambda(1 - \lambda)}$$

$$\pi_{SC}^{I*} = \pi_F^{I*} + \pi_P^{I*} = \frac{1}{4} + \frac{\epsilon^2}{4\lambda(1 - \lambda)}$$

$$CS^I = \int_{\frac{p_r}{\lambda}}^{\frac{p_b - p_r}{1-\lambda}} (\lambda v - p_r)dv + \int_{\frac{p_b - p_r}{1-\lambda}}^{1} (v - p_b)dv = \frac{1}{8} + \frac{\epsilon^2}{8\lambda(1 - \lambda)}$$

$$SW^I = \pi_{SC}^{I*} + CS^I = \frac{3}{8} + \frac{3\epsilon^2}{8\lambda(1 - \lambda)}$$

Proof of Lemma 1

Given the demand function $D_b = 1 - \frac{p_b - p_r}{1-\lambda}$, $D_r = \frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}$, we utilize the backward induction technique and first consider the platform's optimal problem,

$$\max_{p_r} \pi_P = (p_r - w)(\frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}) + \epsilon(\frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda})$$

Calculating the first derivative of p_r, and making it equal to zero, we have: $p_r(w, p_b) = \frac{1}{2}(w - \epsilon + \lambda p_b)$. Then we substitute $p_r(w, p_b)$ into the firm's problem,

$$\max_{w, p_b} \pi_F = p_b(1 - \frac{p_b - p_r}{1-\lambda}) + w(\frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda})$$

We first show that π_F is jointly concave in p_b and w by examining the Hessian.

$$H = \begin{bmatrix} \frac{2-\lambda}{-1+\lambda} & \frac{1}{1-\lambda} \\ \frac{1}{1-\lambda} & \frac{1}{(-1+\lambda)\lambda} \end{bmatrix}$$

It is easy to check that the Hessian matrix is negative definite. Setting the first derivative of p_b and w equal to zero, we get $p_b^{W*} = \frac{1}{2}$ and $w_r^{W*} = \frac{\lambda + \epsilon}{2}$. Substitute $p_b^{W*}$and p_r^{W*} into $p_r(w, p_b)$, and we have $p_r^{W*} = \frac{2\lambda - \epsilon}{4}$. It is easy to check that the results satisfy the consumption $\frac{p_r}{\lambda} < p_b$. Similar to the integrated channel model, we can get other equilibrium results.

Proof of Proposition 1

Comparing equilibrium outcomes under the wholesale model (*W*) with that under the single traditional channel (*N*), we find, $p_b^W - p_b^N = 0$;

$$D_b^W - D_b^N = \frac{-\epsilon}{4(1-\lambda)} < 0,\ D_b^W + D_r^W - D_b^N = \frac{\epsilon}{4\lambda} > 0$$

$$\pi_F^W - \pi_F^N = \frac{\epsilon^2}{8\lambda(1-\lambda)} > 0,\ \pi_{SC}^W - \pi_{SC}^N = \frac{3\epsilon^2}{16\lambda(1-\lambda)} > 0;$$

$$CS^W - CS^N = \frac{\epsilon^2}{32\lambda(1-\lambda)},\ SW^W - SW^N = \frac{7\epsilon^2}{32\lambda(1-\lambda)} > 0.$$

Proof of Proposition 2

Comparing equilibrium outcomes under the wholesale model (*W*) with that under the integrated channel (*I*), we find,

$$p_b^W - p_b^I = 0,\ p_r^W - p_r^I = \frac{\epsilon}{4} > 0;$$

$$D_b^W - D_b^I = \frac{\epsilon}{4(1-\lambda)} > 0,\ D_r^W - D_r^I = -\frac{\epsilon}{4\lambda(1-\lambda)} < 0,\ D_b^W + D_r^W - (D_b^I + D_r^I) = -\frac{\epsilon}{4\lambda} < 0;$$

$$\pi_F^W - \pi_F^I = \frac{\epsilon(\epsilon + 2\alpha\epsilon - 2\alpha\lambda)}{8(-1+\lambda)\lambda},\ \pi_P^W - \pi_P^I = \frac{\epsilon(\epsilon + 4\alpha\epsilon - 4\alpha\lambda)}{16(1-\lambda)\lambda},\ \pi_{SC}^W - \pi_{SC}^I = -\frac{\epsilon^2}{16(1-\lambda)\lambda} < 0;$$

$$CS^W - CS^I = -\frac{3\epsilon^2}{32\lambda(1-\lambda)} < 0,\ SW^W - SW^I = -\frac{5\epsilon^2}{32\lambda(1-\lambda)} < 0$$

Because $\alpha \in (0, \frac{1}{2})$, we can easily get that when $0 < \alpha < \min\{\frac{\epsilon}{2(\lambda - \epsilon)}, \frac{1}{2}\}$, $\pi_F^W < \pi_F^I$, otherwise $\pi_F^W > \pi_F^I$; and when $0 < \alpha < \min\{\frac{\epsilon}{4(\lambda - \epsilon)}, \frac{1}{2}\}$, $\pi_P^W > \pi_P^I$, otherwise $\pi_P^W < \pi_P^I$.

Proof of Lemma 2

Given the demand function $D_b = 1 - \frac{p_b - p_r}{1-\lambda}$, $D_r = \frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}$, the firm determines the retail price p_b and rental price p_r simultaneously and the optimal problem is:

$$\max_{p_b, p_r} \pi_F = p_b(1 - \frac{p_b - p_r}{1-\lambda}) + (1-\alpha)p_r(\frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}) + \epsilon(\frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda})$$

We first calculate the Hessian matrix and get:

$$H = \begin{bmatrix} \frac{-2}{1-\lambda} & \frac{2-\alpha}{1-\lambda} \\ \frac{2-\alpha}{1-\lambda} & \frac{-2(1-\alpha)}{(1-\lambda)\lambda} \end{bmatrix}$$

We find that when $4(1-\alpha) - (2-\alpha)^2\lambda > 0$, π_F is jointly concave in p_b and p_r. Next, we set the first derivative of p_b and p_r equal to zero, and get $p_b^{A*} = \frac{2(1-\alpha)(1-\lambda) - \alpha\epsilon}{4(1-\alpha) - (2-\alpha)^2\lambda}$ and $p_r^{A*} = \frac{\lambda(2-\alpha)(1-\lambda+\epsilon) - 2\epsilon}{4(1-\alpha) - (2-\alpha)^2\lambda}$. From the assumption $\frac{p_r}{\lambda} < p_b$, we get $\frac{(1-\lambda)(-2\epsilon + \alpha\lambda)}{\lambda(4(1-\alpha) - (2-\alpha)^2\lambda)} < 0$, i.e., $\alpha\lambda < 2\epsilon$. Similar to the integrated channel model, we can get other equilibrium results.

Proof of Proposition 3

Comparing equilibrium outcomes under the agency model (A) with that under the single traditional channel (N), we find,

$$p_b^A - p_b^N = \frac{\alpha(-2\epsilon + \alpha\lambda)}{2(4(1-\alpha) - (2-\alpha)^2\lambda)} < 0$$

$$D_b^A - D_b^N = \frac{(2-\alpha)(-2\epsilon + \alpha\lambda)}{2(4(1-\alpha) - (2-\alpha)^2\lambda)} < 0,\ D_b^A + D_r^A - D_b^N = \frac{(2(1-\lambda) + \alpha\lambda)(2\epsilon - \alpha\lambda)}{2\lambda(4(1-\alpha) - (2-\alpha)^2\lambda)} > 0$$

$$\pi_F^A - \pi_F^N = \frac{(-2\epsilon + \alpha\lambda)^2}{4\lambda(4(1-\alpha) - (2-\alpha)^2\lambda)} > 0,\ \pi_{SC}^A - \pi_{SC}^N = \frac{(-2\epsilon + \alpha\lambda)^2}{4\lambda(4(1-\alpha) - (2-\alpha)^2\lambda)} + \pi_P^A > 0$$

$$CS^A - CS^N = \frac{(2\epsilon - \alpha\lambda)(2\epsilon(4 + (-4+\alpha^2)\lambda) + \alpha\lambda(4 - 4\lambda + \alpha(-8 + (8-3\alpha)\lambda)))}{8\lambda(4(1-\alpha) - (2-\alpha)^2\lambda)^2}$$

Because $\alpha\lambda < 2\epsilon$, $2\epsilon(4 + (-4+\alpha^2)\lambda) + \alpha\lambda(4 - 4\lambda + \alpha(-8 + (8-3\alpha)\lambda)) > \alpha\lambda(4 + (-4+\alpha^2)\lambda) + \alpha\lambda(4 - 4\lambda + \alpha(-8 + (8 - 3\alpha)\lambda)) = 2\alpha\lambda(4(1-\alpha) - (2-\alpha)^2\lambda) > 0$.

Proof of Proposition 4

Comparing the profit of the firm under the agency model (A) with that under the wholesale model (W), we find,

$$\Delta\pi_F = \pi_F^A - \pi_F^W = \frac{\alpha^2(\lambda\epsilon^2 + 2(1-\lambda)\lambda^2) + 4\alpha(1-\lambda)\epsilon(\epsilon - 2\lambda) + 4(1-\lambda)\epsilon^2}{8\lambda(1-\lambda)(4(1-\alpha) - (-2+\alpha)^2\lambda)}$$

We define $f(\alpha) = \alpha^2(\lambda\epsilon^2 + 2(1-\lambda)\lambda^2) + 4\alpha(1-\lambda)\epsilon(\epsilon - 2\lambda) + 4(1-\lambda)\epsilon^2$. If $\Delta = (4(1-\lambda)\epsilon(\epsilon - 2\lambda))^2 - 4(\lambda\epsilon^2 + 2(1-\lambda)\lambda^2)4(1-\lambda)\epsilon^2 = 16\epsilon^2(1-\lambda)((1-2\lambda)\epsilon^2 - 4(1-\lambda)\lambda\epsilon + 2(1-\lambda)\lambda^2) \le 0$, i.e., $g(\epsilon) = (1-2\lambda)\epsilon^2 - 4(1-\lambda)\lambda\epsilon + 2(1-\lambda)\lambda^2 \le 0$, we have $f(\alpha) \ge 0$, $\pi_F^A > \pi_F^W$. As $\partial g(\epsilon)/\partial\epsilon = 2((1-2\lambda)\epsilon - 2(1-\lambda)\lambda) < 2((1-2\lambda)\lambda - 2(1-\lambda)\lambda) < 0$, $g(0) = 2(1-\lambda)\lambda^2 > 0$, there must exist a unique ϵ^* that satisfies $g(\epsilon^*) = 0$. $g(\epsilon) \le 0$ indicates that $\epsilon \ge \epsilon^*$.

If $g(\epsilon) > 0$, i.e., $\epsilon < \epsilon^*$. let $f(\alpha) = 0$, and we have $\alpha_1 = \frac{2(1-\lambda)\epsilon}{(1-\lambda)(2\lambda - \epsilon) + \sqrt{(1-\lambda)g(\epsilon)}}$, $\alpha_2 = \frac{2(1-\lambda)\epsilon}{(1-\lambda)(2\lambda - \epsilon) - \sqrt{(1-\lambda)g(\epsilon)}}$. Because $\frac{\partial\Delta\pi_F}{\partial\alpha} = \frac{(2\epsilon - \alpha\lambda)(-(-2+\alpha)(-1+\lambda)\lambda + \epsilon(2 + (-2+\alpha)\lambda))}{\lambda(4(-1+\alpha) + (-2+\alpha)^2\lambda)^2} < 0$ and $\alpha \in (0, \frac{1}{2})$, we get that when $\alpha_1 \ge \frac{1}{2}$, $f(\alpha) > 0$ for $\alpha \in (0, \frac{1}{2})$, and when $\alpha_1 < \frac{1}{2}$, $f(\alpha) > 0$ *for* $\alpha \in (0, \alpha_1)$ *and* $f(\alpha) < 0$ *for* $\alpha \in (\alpha_1, \frac{1}{2})$.

Proof of Proposition 5

Comparing the platform's profit under the agency model (A) with that under the wholesale model (W), we find,

$$\Delta\pi_P = \pi_P^A - \pi_P^W = \frac{-((4(1-\alpha) - (2-\alpha)^2\lambda)^2 + 32(1-\lambda)\alpha(2(1-\lambda) + \alpha\lambda))\epsilon^2 + 16\alpha(1-\lambda)\lambda(4(1-\lambda) + \alpha^2\lambda)\epsilon - 16(2-\alpha)\alpha^2(1-\lambda)^2\lambda^2}{16\lambda(1-\lambda)(4(1-\alpha) - (2-\alpha)^2\lambda)^2}$$

We define $l(\epsilon) = -((4(1-\alpha) - (2-\alpha)^2\lambda)^2 + 32(1-\lambda)\alpha(2(1-\lambda) + \alpha\lambda))\epsilon^2 + 16\alpha(1-\lambda)\lambda(4(1-\lambda) + \alpha^2\lambda)\epsilon - 16(2-\alpha)\alpha^2(1-\lambda)^2\lambda^2$, and calculate $\Delta = 64\alpha^2(2+\alpha)(1-\lambda)^2\lambda^2(4(1-\alpha) - (2-\alpha)^2\lambda)^2 > 0$.

Let $l(\epsilon) = 0$, and we have $\epsilon_1 = \frac{4\alpha\lambda(1-\lambda)(2(4(1-\lambda) + \alpha^2\lambda) - m\sqrt{\alpha+2})}{m^2 + 32\alpha(1-\lambda)n}$ and $\epsilon_2 = \frac{4\alpha\lambda(1-\lambda)(2(4(1-\lambda) + \alpha^2\lambda) + m\sqrt{\alpha+2})}{m^2 + 32\alpha(1-\lambda)n}$ ($m = 4(1-\alpha) - (2-\alpha)^2\lambda$, $n \equiv 2(1-\lambda) + \alpha\lambda$). From $p_r^A > 0$, $D_b^A > 0$ and $\frac{p_r^A}{\lambda} < p_b^A$, we get $\frac{\alpha\lambda}{2} < \epsilon < min\{\frac{(2-\alpha)\lambda(1-\lambda)}{n}, \frac{(3-\alpha)\alpha\lambda + 2(1-\alpha-\lambda)}{2-\alpha}\}$. We define $\bar{\epsilon} = min\{\frac{(2-\alpha)\lambda(1-\lambda)}{n}, \frac{(3-\alpha)\alpha\lambda + 2(1-\alpha-\lambda)}{2-\alpha}\}$. It is easy to test that $\frac{\alpha\lambda}{2} < \epsilon_1$. We get that if $\epsilon_1 < \bar{\epsilon} \le \epsilon_2$, then $l(\epsilon) < 0$ for $\epsilon \in (\frac{\alpha\lambda}{2}, \epsilon_1)$ and $l(\epsilon) > 0$ for $\epsilon \in (\epsilon_1, \bar{\epsilon})$, and if $\bar{\epsilon} > \epsilon_2$, then $l(\epsilon) < 0$ for $\epsilon \in (\frac{\alpha\lambda}{2}, \epsilon_1)$ and $\epsilon \in (\epsilon_2, \bar{\epsilon})$, and $l(\epsilon) > 0$ for $\epsilon \in (\epsilon_1, \epsilon_2)$.

Proof of Proposition 6

We first calculate the equilibrium results under the wholesale model. When considering rental enthusiastic consumers, the demand of the retail market and rental market are respectively given as: $D_b = \theta(1 - \frac{p_b - p_r}{1-\lambda})$, $D_r = \theta\left(\frac{p_b - p_r}{1-\lambda} - \frac{p_r}{\lambda}\right) + (1-\theta)(1 - \frac{p_r}{\lambda})$. We summarize the equilibrium results in the following Lemma. The calculation process is similar to that of Lemma 1.

Lemma 3. *Under the wholesale model, the optimal solutions are as follows:*

a) *Pricing:* $w^r = \frac{\epsilon + \lambda}{2}$, $p_b^{Wr} = \frac{1}{2}$, $p_r^{Wr} = \frac{(3 + \epsilon - (1+\epsilon)\theta)\lambda - \epsilon - 3(1-\theta)\lambda^2}{4 - 4(1-\theta)\lambda}$.

b) *Demand:* $D_b^{Wr} = \frac{\theta(\epsilon(1-\theta)\lambda + (1-\lambda)(2 - (1-\theta)\lambda) - \epsilon)}{4(1-\lambda)(1-(1-\theta)\lambda)}$, $D_r^{Wr} = \frac{1}{4}(1 - \theta + \frac{\epsilon\theta}{1-\lambda} + \frac{\epsilon}{\lambda})$.

c) *Profits:* $\pi_F^{Wr} = \frac{1}{8}(2\epsilon(1-\theta) + \epsilon^2(\frac{\theta}{1-\lambda} + \frac{1}{\lambda}) + \lambda + \theta(1 - \lambda + \frac{1}{1-(1-\theta)\lambda}))$,

$$\pi_P^{Wr} = \frac{(\epsilon - \epsilon(1-\theta)\lambda + (1-\theta)(1-\lambda)\lambda)^2}{16(1-\lambda)\lambda(1-(1-\theta)\lambda)}$$

$$\pi_{SC}^{Wr} = \frac{1}{16}(6 \epsilon (1-\theta) + 3 \epsilon^2 (\frac{\theta}{1-\lambda} + \frac{1}{\lambda}) + 3\lambda + \theta(3 - 3\lambda + \frac{1}{1-(1-\theta)\lambda}))$$

Next, we compare the firm's profit under wholesale contract with the rental enthusiastic consumer market (*Wr*) and that under the single traditional channel (*N*) and find,

$$\Delta\pi_F = \pi_F^{Wr} - \pi_F^N = \frac{1}{8}(-2 + 2 \epsilon (1-\theta) + \epsilon^2 (\frac{\theta}{1-\lambda} + \frac{1}{\lambda}) + \lambda + \theta(1 - \lambda + \frac{1}{1-(1-\theta)\lambda}))$$

We define $\theta_{W1}^r \equiv \frac{(1-\lambda)\lambda(\epsilon (1-2\lambda) - \epsilon^2 - (1-\lambda)^2) - \sqrt{(2 \epsilon^2 + 2 \epsilon (-1+\lambda) + (-1+\lambda)^2)(-1+\lambda)^2\lambda^2}}{\lambda^2(1 - \epsilon - \lambda)^2}$ and $\theta_{W2}^r \equiv \frac{\sqrt{(2 \epsilon^2 + 2 \epsilon (-1+\lambda) + (-1+\lambda)^2)(-1+\lambda)^2\lambda^2} - (1-\lambda)\lambda(\epsilon^2 + (1-\lambda)^2 - \epsilon (1-2\lambda))}{\lambda^2(1 - \epsilon - \lambda)^2}$. It is easy to prove $\theta_{W1}^r < \theta_{W2}^r < 1$. If $\theta_{W2}^r > 0$, i.e., $\theta_{W2}^r = 0.8$, then for all $\min\{\theta_{W1}^r, 0\} < \theta < \theta_{W2}^r$, we have $\pi_F^{Wr} < \pi_F^N$; besides, for $0 < \theta < \min\{\theta_{W1}^r, 0\}$ and $\theta_{W2}^r < \theta < 1$, we have $\pi_F^{Wr} > \pi_F^N$.

If $\theta_{W2}^r < 0$, then for all the market, we have $\pi_F^{Wr} > \pi_F^N$.

Proof of Proposition 7

We first calculate the equilibrium results under agency contract and summarize in the following Lemma. The proofs are similar to that of Lemma 2. For convenience, we define $m^r \equiv 4(1-\lambda) - \alpha(4 - (4 - \alpha\theta)\lambda)$.

Lemma 4. *Under the agency model, the optimal solutions are as follows:*

a) *Pricing*: $p_b^{Ar} = \frac{2(1-\lambda) - \alpha(\epsilon - \epsilon (1-\theta)\lambda + (1-\lambda)(2 + (1-\alpha)(1-\theta)\lambda))}{m^r}$,

$$p_r^{Ar} = \frac{(2 - \alpha(2-\theta))(1-\lambda)\lambda - \epsilon(2 - (2 - \alpha\theta)\lambda)}{m^r}$$

b) *Demand*: $D_b^{Ar} = \frac{\theta(2(1 - \epsilon - \lambda) - \alpha(2 - \epsilon - (3-\alpha)\lambda))}{m^r}$,

$$D_r^{Ar} = \frac{2 \epsilon + 2(1 - \alpha - \epsilon)\lambda - (2 - \alpha - 2 \epsilon)\theta\lambda - 2(1-\alpha)(1-\theta)\lambda^2}{\lambda m^r}$$

c) *Profits*: $\pi_F^{Ar} = \frac{(1-\alpha)(1-\lambda)\lambda(\theta + (1-\alpha)(1-\theta)\lambda) + \epsilon \lambda(2 - 2\alpha - 2\theta + \alpha\theta - 2(1-\alpha)(1-\theta)\lambda) + \epsilon^2 (1 - \lambda + \theta\lambda)}{\lambda m^r}$,

$$\pi_P^{Ar} = \frac{\alpha((2 - \alpha - 2 \epsilon)\theta\lambda + 2(1-\alpha)(1-\theta)\lambda^2 - 2 \epsilon - 2(1 - \alpha - \epsilon)\lambda)((\alpha(2-\theta) - 2)(1-\lambda)\lambda + \epsilon (2 - (2 - \alpha\theta)\lambda))}{\lambda(m^r)^2}$$

Next, we compare the firm's profit under the agency contract with the rental enthusiastic consumer market (*Ar*) and that under the single traditional channel (*N*) and find,

$$\Delta\pi_F = \pi_F^{Ar} - \pi_F^N = \frac{(1-\alpha)(1-\lambda)\lambda(\theta + (1-\alpha)(1-\theta)\lambda) + \epsilon \lambda(2 - 2\alpha - 2\theta + \alpha\theta - 2(1-\alpha)(1-\theta)\lambda) + \epsilon^2 (1 - \lambda + \theta\lambda)}{\lambda m^r} - \frac{1}{4}$$

We define $\theta_A^r \equiv \frac{4(-1+\lambda)(\epsilon^2 - \lambda + \alpha\lambda + 2 \epsilon \lambda - 2\alpha \epsilon \lambda + \lambda^2 - 2\alpha\lambda^2 + \alpha^2\lambda^2)}{\lambda(4 - 4\alpha - 8 \epsilon + 4\alpha \epsilon + 4 \epsilon^2 - 8\lambda + 12\alpha\lambda - 3\alpha^2\lambda + 8 \epsilon \lambda - 8\alpha \epsilon \lambda + 4\lambda^2 - 8\alpha\lambda^2 + 4\alpha^2\lambda^2)}$. In the agency model, for $\frac{1}{2-\alpha} < \lambda < \frac{4-4\alpha}{4 - 4\alpha + \alpha^2\theta}$, $\frac{2\lambda - 2\alpha\lambda - 2\theta\lambda + \alpha\theta\lambda - 2\lambda^2 + 2\alpha\lambda^2 + 2\theta\lambda^2 - 2\alpha\theta\lambda^2}{2(1 - \lambda + \theta\lambda)} < \epsilon < \frac{-2 + 2\alpha + 2\lambda - 3\alpha\lambda + \alpha^2\lambda}{-2+\alpha}$, or $0 < \lambda < \frac{1}{2-\alpha}$, $-\frac{2\lambda - 2\alpha\lambda - 2\theta\lambda + \alpha\theta\lambda - 2\lambda^2 + 2\alpha\lambda^2 + 2\theta\lambda^2 - 2\alpha\theta\lambda^2}{2(1 - \lambda + \theta\lambda)} < \epsilon < \frac{-(2 - 2\alpha + \alpha\theta)(-1+\lambda)\lambda}{2 - 2\lambda + \alpha\theta\lambda}$, if $0 < \theta_A^r < 1$, for $\theta_A^r < \theta < 1$, we have $\pi_F^{Ar} > \pi_F^N$; besides, we have $\pi_F^{Ar} < \pi_F^N$.

If $\theta_A^r < 0$, then for all the market, we have $\pi_F^{Ar} > \pi_F^N$. Otherwise, $\theta_A^r > 1$, for all the market, we have $\pi_F^{Ar} < \pi_F^N$.

Proof of Proposition 8

To proof Proposition 8, we first calculate the equilibrium results under different pricing models. We use superscripts "*Nc*", "*Wc*" and "*Ac*" to represent equilibrium results under the single channel, the wholesale model and the agency model, respectively. Under the single traditional channel, considering consumers' conspicuous behavior, consumers' utility from buying the product is $u_b = v - p_b - \delta D_b$, and we get the demand function is $D_b^{Nc} = \frac{1-p_b}{1+\delta}$. Similar to the proof of the single traditional channel in the base model, we get $p_b^{Nc} = \frac{1}{2}$, $D_b^{Nc} = \frac{1}{2(1+\delta)}$, $\pi_F^{Nc} = \frac{1}{4(1+\delta)}$.

With the presence of a rental platform, consumers' utility from buying the product is $u_b = v - p_b - \delta(D_b + D_r)$, and the utility from renting the product is $u_r = \lambda v - p_r$. We set $u_b = u_r$, and get $v^* = \frac{-\delta\lambda - \lambda p_b + \delta p_r + \lambda p_r}{(-1+\lambda)\lambda}$. The demand of the retail market and rental market are respectively given as:

$$D_b = 1 - \frac{-\delta\lambda - \lambda p_b + \delta p_r + \lambda p_r}{(-1+\lambda)\lambda},\ D_r = \frac{-\delta\lambda - \lambda p_b + \delta p_r + \lambda p_r}{(-1+\lambda)\lambda} - \frac{p_r}{\lambda}$$

We summarize the equilibrium outcomes in the following Lemmas.

Lemma 5. *When considering conspicuous behavior, under condition* $8(1-\lambda)\lambda-\delta^2>0$*, the optimal solutions under the wholesale model are as follows:*

a) *Pricing*:

$$w^c=\frac{\delta^3(\epsilon+\lambda)+4(-1+\lambda)\lambda(\epsilon+\lambda)+\delta^2(\epsilon+\epsilon\lambda+\lambda^2)+\delta\lambda(2(-1+\lambda)(1+2\lambda)+\epsilon(-3+4\lambda))}{(1+\delta)(\delta^2+8(-1+\lambda)\lambda)}$$

$$p_b^{Wc}=\frac{\delta\epsilon+4(-1+\lambda)\lambda+\delta^2(\epsilon+\lambda)}{\delta^2+8(-1+\lambda)\lambda},\ p_r^{Wc}=\frac{\lambda(\delta^3+\delta\epsilon(3-2\lambda)-2(\epsilon-2\lambda)(-1+\lambda)+\delta^2(\epsilon+\lambda)+\delta(-1+\lambda)(1+6\lambda))}{(1+\delta)(\delta^2+8(-1+\lambda)\lambda)}$$

b) *Demand*:

$$D_b^{Wc}=\frac{\delta(1+\delta)\epsilon+(-4-\delta+\delta^2+2(1+\delta)\epsilon)\lambda+2(2+\delta)\lambda^2}{(1+\delta)(\delta^2+8(-1+\lambda)\lambda)},\ D_r^{Wc}=\frac{\delta-2\epsilon-2\delta\epsilon-2\delta\lambda}{\delta^2+8(-1+\lambda)\lambda}$$

c) *Profits*:

$$\pi_F^{Wc}=\frac{-\epsilon^2+\delta\epsilon(1-2\epsilon-2\lambda)+2(-1+\lambda)\lambda-\delta^2(-1+\epsilon+\lambda)(\epsilon+\lambda)}{(1+\delta)(\delta^2+8(-1+\lambda)\lambda)},\ \pi_P^{Wc}=-\frac{(-1+\lambda)\lambda(2\epsilon+\delta(-1+2\epsilon+2\lambda))^2}{(1+\delta)(\delta^2+8(-1+\lambda)\lambda)^2}$$

Lemma 6. *When considering conspicuous behavior, under the condition* $\delta^2-4\lambda+2\alpha(2+\delta)\lambda+(-2+\alpha)^2\lambda^2<0$*, the optimal solutions under the agency model are as follows:*

a) *Pricing*:

$$p_b^{Ac}=\frac{\delta(1+\delta)\epsilon+(-2+\delta^2+\alpha(2+\epsilon+\delta(-\delta+\epsilon)))\lambda-(-1+\alpha)(2+\alpha\delta)\lambda^2}{\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2}$$

$$p_r^{Ac}=\frac{\lambda(\delta^2+2\epsilon+(-2+\alpha)\lambda+(-2+\alpha)(\epsilon-\lambda)\lambda+\delta(-1+\epsilon+\lambda+\alpha\lambda))}{\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2}$$

b) *Demand*:

$$D_b^{Ac}=\frac{2(-1+\alpha)\lambda+(\delta-(-2+\alpha)\lambda)(\epsilon+\lambda-\alpha\lambda)}{\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2},\ D_r^{Ac}=\frac{\delta-2\epsilon-2\delta\epsilon+(\alpha+2(-1+\alpha)\delta)\lambda}{\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2}$$

c) *Profits*:

$$\pi_F^{Ac}=\frac{(-1+\alpha)\lambda-(\epsilon+\lambda-\alpha\lambda)(\epsilon-\lambda+\delta(-1+\epsilon+\lambda-\alpha\lambda))}{\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2}$$

$$\pi_P^{Wc}=\frac{\alpha\lambda(\delta-2\epsilon-2\delta\epsilon+(\alpha+2(-1+\alpha)\delta)\lambda)(\delta^2+2\epsilon+(-2+\alpha)\lambda+(-2+\alpha)(\epsilon-\lambda)\lambda+\delta(-1+\epsilon+\lambda+\alpha\lambda))}{(\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2)^2}$$

Next, we compare the firm's profit under the wholesale and agency contract with that under the single traditional channel and find:

$$\pi_F^{Wc}-\pi_F^{Nc}=-\frac{(2\epsilon+\delta(-1+2\epsilon+2\lambda))^2}{4(1+\delta)(\delta^2+8(-1+\lambda)\lambda)}>0$$

$$\pi_F^{Ac}-\pi_F^{Nc}=-\frac{(\delta-2\epsilon-2\delta\epsilon+(\alpha+2(-1+\alpha)\delta)\lambda)^2}{4(1+\delta)(\delta^2+2(-2+\alpha(2+\delta))\lambda+(-2+\alpha)^2\lambda^2)}>0$$

Proof of Proposition 10

We first calculate the equilibrium results under the wholesale contract. In the two-period rental model, the profit function of the firm and the platform are respectively given as:

$$\pi_F=p_b(D_{1b}+D_{2b})+wD_{2r},\ \pi_P=p_r(D_{1r}+D_{2r})-wD_{2r}+\epsilon D_{2r}$$

We summarize the equilibrium results under the wholesale contract in the following lemma. We use superscript ij to represent the players choose j pricing strategy (low price difference strategy denoted by L and high price difference strategy denoted by H) under different contract (wholesale contract denoted by W and agency contract denoted by A). The proofs are similar to that of Lemma 1.

Lemma 7. *Under the wholesale model, the equilibrium of high price difference strategy doesn't exist, the optimal solutions of low price difference strategy are as follows:*

a) *Pricing*:

$$w^{WL} = \frac{\epsilon\,(2-3\lambda)^2(2-\lambda)(1-\lambda) + 4\lambda(2+(-4+\lambda)\lambda)^2}{2(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

$$p_b^{WL} = \frac{(1-\lambda)^3(8+(\epsilon+4(-4+\lambda))\lambda)}{2(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))},\ p_r^{WL} = \frac{(1-\lambda)^2(-2\,\epsilon+(8+5\,\epsilon)\lambda-2(9+\epsilon)\lambda^2+8\lambda^3)}{2(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

b) *Demand*:

$$D_{1b}^{WL} = \frac{8-\epsilon(1-\lambda)(2-(4-\lambda)\lambda)+2\lambda(-16+\lambda(21+2(-5+\lambda)\lambda))}{2(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

$$D_{2b}^{WL} = \frac{2(1-2\lambda)(4+\lambda(\lambda(33+\lambda(-21+5\lambda))-20))-\epsilon\,(-1+\lambda)^2(2+(-4+\lambda)\lambda)}{2(1-2\lambda)(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

$$D_{1r}^{WL} = \frac{(1-\lambda)(\epsilon\,(2-\lambda(5+(-3+\lambda)\lambda))-2\lambda^2(3+2(-3+\lambda)\lambda))}{2(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

$$D_{2r}^{WL} = \frac{(1-\lambda)^3(2(1-2\lambda)\lambda^2+\epsilon\,(1-\lambda)(2-3\lambda))}{2\lambda(1-2\lambda)(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

c) *Profits:*

$$\pi_F^{WL} = \frac{(1-\lambda)^3(4\,\epsilon\,(1-2\lambda)\lambda^2+\epsilon^2\,(1-\lambda)(2-3\lambda)+8\lambda(1-2\lambda)(2+(-4+\lambda)\lambda))}{4\lambda(1-2\lambda)(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))}$$

$$\pi_P^{WL} = \frac{(1-\lambda)^3(4\lambda^2(\epsilon\,(2-(5-\lambda)\lambda)-\lambda(8-(17-6\lambda)\lambda))(1-2\lambda)(2-(4-\lambda)\lambda) + \epsilon^2\,(8-56\lambda+158\lambda^2-228\lambda^3+175\lambda^4-67\lambda^5+11\lambda^6))}{4\lambda(1-2\lambda)(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))^2}.$$

Without the rental platform, the demands of the firm in two periods are $D_{1b}^N = 1 - p_b$ and $D_{2b}^N = 1 - \frac{p_b}{1-\lambda}$, respectively. Similar to the proof of the single traditional channel in the base model, we get $p_b^N = \frac{1-\lambda}{2-\lambda}$, $D_{1b}^N = \frac{1}{2-\lambda}$, $D_{2b}^N = \frac{1-\lambda}{2-\lambda}$ and $\pi_F^N = \frac{1-\lambda}{2-\lambda}$.

Next, we compare the firm's profit under the wholesale contract with that under the single traditional channel and find:

$$\pi_F^{WL} - \pi_F^N = \frac{4\,\epsilon\,(2-\lambda)(1-\lambda)^3\lambda^2(1-2\lambda)+\epsilon^2\,(2-\lambda)(1-\lambda)^4(2-3\lambda)+4(1-\lambda)\lambda^3(1-2\lambda)(4-\lambda(11-2(4-\lambda)\lambda))}{4\lambda(1-2\lambda)(8-(2-\lambda)\lambda(18-\lambda(19-8\lambda)))(2-\lambda)} > 0$$

Proof of Proposition 11

In lemma 7, we show that the equilibrium of the high price difference strategy doesn't exist under the wholesale contract. Next, we calculate the equilibrium results of two pricing strategies under the agency contract. The profit functions can be expressed as:

$$\pi_F = p_b(D_{1b}+D_{2b}) + (1-\alpha)p_r(D_{1r}+D_{2r}) + \epsilon\,D_{2r},\ \pi_P = \alpha p_r(D_{1r}+D_{2r})$$

We summarize the equilibrium results of different pricing strategies in the following lemmas.

Lemma 8. *When the firm chooses the low price difference strategy, the optimal solutions are as follows:*

a) *Pricing*:

$$p_b^{AL} = \frac{(1-\lambda)(2(1-2\lambda)(4+\lambda(-8+\epsilon+2\lambda))+\alpha(4(2\lambda-1)(2+(\lambda-4)\lambda)+\epsilon(\lambda(3+\lambda)-2)))}{(2-3\lambda)(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)}$$

$$p_r^{AL} = \frac{(1-\lambda)(2\lambda(2-\alpha)(1-2\lambda)-\epsilon(2-(4-\alpha)\lambda))}{8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2}$$

b) *Demand*:

$$D_{1b}^{AL} = \frac{4(1-\lambda)(1-2\lambda)(2-\epsilon-2\lambda)+\alpha(1-2\lambda)(\epsilon\,(2-\lambda)+2\lambda(10-7\lambda)-8)-\alpha^2(2-3\lambda)^2\lambda}{(2-3\lambda)(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)}$$

$$D_{2b}^{AL} = \frac{4(2-\lambda)\lambda(\epsilon+8\lambda)+4(2-\epsilon-10\lambda)+\alpha(1-\lambda)(\epsilon(2-\lambda)+36\lambda-38\lambda^2-8)-\alpha^2(2-3\lambda)^2\lambda}{(2-3\lambda)(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)}$$

$$D_{1r}^{AL} = \frac{\epsilon(4-14\lambda+14\lambda^2-4\lambda^3+\alpha\lambda^3)-2\lambda(1-2\lambda)(2(1-\lambda)\lambda+\alpha(2-\lambda)(1-2\lambda))}{\lambda(2-3\lambda)(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)}$$

$$D_{2r}^{AL} = \frac{2(1-\lambda)(\lambda^2(2-4\lambda+\alpha(3+\lambda))+\epsilon(2-\lambda(5-(4-\alpha)\lambda))-2\alpha\lambda)}{\lambda(2-3\lambda)(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)}$$

c) *Profits*:

$$\pi_F^{AL} = \frac{(1-\lambda)(4(1-\alpha)\lambda(1-2\lambda)(2-(4-\lambda)\lambda)+\epsilon^2(2-\lambda(5-(4-\alpha)\lambda))+2\epsilon\lambda(2(1-2\lambda)\lambda+\alpha(\lambda(3+\lambda)-2)))}{\lambda(2-3\lambda)(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)}$$

$$\pi_P^{AL} = \frac{\alpha(1-\lambda)(\epsilon(2-(4-\alpha)\lambda)-2(2-\alpha)\lambda(1-2\lambda))(2\alpha\lambda(2-(4-\lambda)\lambda)-\epsilon(4-\lambda(8-(4-\alpha)\lambda)))}{\lambda(8-8\alpha-2(12-(12-\alpha)\alpha)\lambda+(4-\alpha)(4-3\alpha)\lambda^2)^2}.$$

Lemma 9. *When the firm chooses the high price difference strategy, the optimal solutions are as follows:*

d) *Pricing*:

$$p_b^{AH} = \frac{(1-\lambda)(4-2\epsilon-\alpha(4-\epsilon+(1-\alpha)\lambda))}{8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda}$$

$$p_r^{AH} = \frac{(1-\lambda)((4-3\alpha)\lambda-2\epsilon)}{8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda}$$

e) *Demand*:

$$D_{1b}^{AH} = \frac{4(1-\lambda)-\alpha(4+\epsilon-2(3-\alpha)\lambda)}{8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda} D_{2b}^{AH} = 0,$$

$$D_{1r}^{AH} = \frac{\epsilon(2(1-\lambda)+\alpha\lambda)-\alpha\lambda(1+\lambda-\alpha\lambda)}{\lambda(8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda)}, D_{2r}^{AH} = 1 - \frac{(1-\lambda)((4-3\alpha)\lambda-2\epsilon)}{\lambda(8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda)}$$

f) *Profits*:

$$\pi_F^{AH} = \frac{\epsilon^2(1-\lambda)+2(1-\alpha)(1-\lambda)\lambda(1+(1-\alpha)\lambda)+\epsilon\lambda(4-5\alpha-(4-\alpha)(1-\alpha)\lambda)}{\lambda(8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda)}$$

$$\pi_P^{AH} = \frac{\alpha(1-\lambda)((4-3\alpha)\lambda-2\epsilon)(\epsilon(4(1-\lambda)+\alpha\lambda)+2\lambda(2-3\alpha-2(1-\alpha)\lambda))}{\lambda(8(1-\alpha)-(8(1-\alpha)+\alpha^2)\lambda)^2}.$$

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